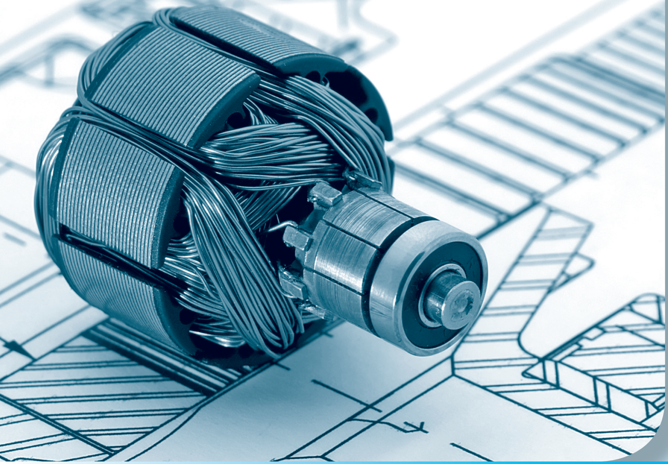

6

DC Machines

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What You Will Learn in This Chapter

A Theoretical Highlights

1. The principles of motor and generator operation
2. Flux-density distribution and armature reaction
3. Types of dc machines and their characteristics
4. Control of starting current
5. Compound generators
6. Modern methods of speed control

B Outstanding Practical Characteristics

1. Principles of operation and motor-generator characteristics
2. Effects of armature reaction on the machines' performances
3. Control of the torque-speed characteristics of series and separately excited dc motors
4. How to minimize the motor's high starting current
5. Thyristor speed control of dc motors
6. Power factor of thyristor-controlled dc motor
7. Typical manufacturers' machine characteristics

C Additional Students' Aid on the Web

1. Transient analysis
2. Time-constant considerations
3. Block diagrams and transfer functions
4. Transformation of mechanical parameters
5. General machine equations
6. Examples, exercises, and problems

6.0 Introduction

Direct-current motors are widely used in hoists, steel mills, cement plants, the pulp and paper industry, and mining operations. They are versatile and can be adjusted to meet tough job requirements, such as high starting torque, constant-power or constant-torque loads, quick acceleration and deceleration, and speed variations of up to $\pm$ 400% of base value.

Although their torque-speed characteristics are easily adjustable, dc machines are not used for ordinary industrial applications because of the limitations, cost, and unavailability of dc voltage sources.

This chapter deals with the dc machine's principles of operation, steady-state operation, and modern techniques of speed control.

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6.1 Steady-State Analysis

6.1.1 General

Like all the three-phase ac machines, direct-current machines have a stator and a rotor winding. The stator winding is referred to as the field winding, and the rotor winding is known as the armature winding. Manufacturer's pictures of a dc motor and its stator and rotor windings are shown in Figs. 6-1, 6-2, and 6-3.

FIG. 6-1 A 375 kW dc motor. *Courtesy of General Electric*

FIG. 6-2 Stator of a 375 kW dc motor. *Courtesy of General Electric*

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FIG. 6-3 Rotor of a 1200 kW, 600 V, 900 r/min dc motor. *Courtesy of General Electric*

Field windings are wound around the poles of the stator and are supplied with dc current, which produces the main magnetic field of the machine. Depending on the machine type and rating, the stator structure may incorporate additional windings, such as the compensating, the commutating, and the auxiliary field windings. The functions and the relative physical location of these windings are described in Sections 6.1.5 and 6.1.7.

Armature windings are placed in the rotor slots, which are uniformly distributed around the rotor's periphery. The end connections of the armature windings terminate in the commutator segments (longitudinal bars of copper), which ride on the stationary brushes. The commutator segments are insulated from each other. In conjunction with the brushes, they rectify the ac current of the armature windings. The armature current of a dc motor is conducted from the external machine terminals to the brushes, then to the commutator segments, and from there to the armature conductors.

Depending on how the field winding is excited relative to the armature coil, dc machines are generally classified as shunt, series, or separately excited. As shown in Fig. 6-4, the field winding in a shunt machine is connected in parallel to the armature.

In series dc machines, the field winding is connected in series with the armature winding (see Fig. 6-14(a)). Depending on the location and relative magnetic orientation of the auxiliary field winding, dc shunt machines are further subdivided into other categories (see Section 6.1.7).

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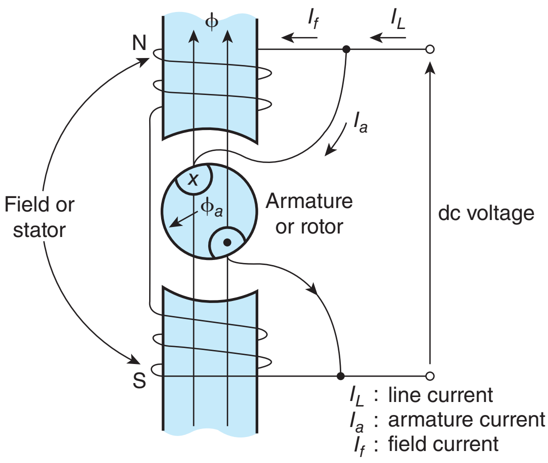

FIG. 6-4 An elementary representation of a two-pole dc shunt motor. (The effect of the armature flux on the field flux is neglected.)

Regardless of the winding connection, as shown in Fig. 6-4, the current through the field windings (I_f) produces the field flux (ϕ_f), and the current through the armature windings (I_a) produces the armature flux (ϕ_a). The field flux combines with the armature flux to produce the machine's net effective flux (ϕ) per pole. The natural tendency of the stator and rotor fields to align their magnetic axes produces torque.

6.1.2 Principles of Operation

The equation that describes the generator-motor operation can be derived from the concept of the speed voltage (Eq. (1.139)) and Ampere's law (Eq. (1.194)), or from use of the concepts associated with the energy stored in a coil and the definition of flux linkages. This section uses the last-named method because it gives a better understanding of the fundamentals.

Motor Principles

The operation of a dc motor is based on the tendency of a current-carrying coil, when placed in an external magnetic field, to rotate in such a way that the field it creates is parallel to, and in the same direction as, the external field. The general expression for the torque developed by a dc motor is derived as follows.

For the sake of simplicity, the armature winding shown in Fig. 6-5 has only a single-loop coil. The instantaneous value of the torque developed (T_i) is

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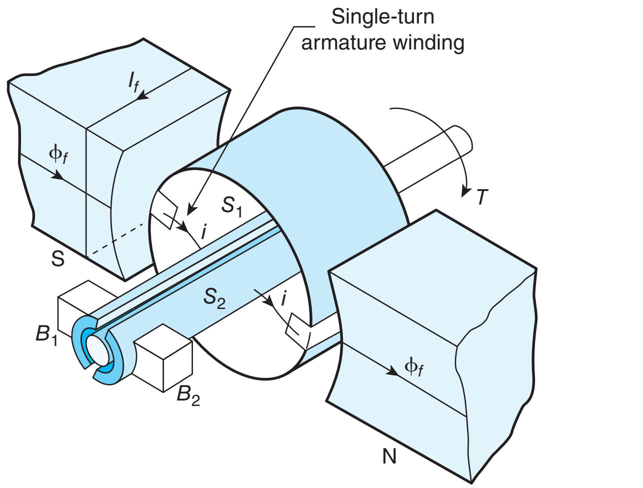

FIG. 6-5 An elementary representation of a dc machine.

determined by the partial derivative of the energy stored (W_f) in the armature winding, with respect to its angle of rotation (θ). Mathematically expressed,

$$T_i = \frac{\partial}{\partial \theta} W_f \qquad \textbf{(6.1)}$$

The incremental energy stored is given by

$$dW_f = v_g i\, dt \qquad \textbf{(6.2)}$$

where v_g is the voltage induced in a single turn of the armature coil and i is its current. From the above, we obtain

$$T_i = \frac{v_g i\, dt}{\partial \theta} \qquad \textbf{(6.3)}$$

If we substitute for the voltage induced its equivalent expression in terms of the flux linkage (λ), we get

$$T_i = \frac{1}{\partial \theta}\left(i \frac{d\lambda}{dt} dt\right) \qquad \textbf{(6.4)}$$

$$T_i = i \frac{d\lambda}{\partial \theta} \qquad \textbf{(6.5)}$$

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As the rotor turns dx of a revolution, the incremental change in the angle of rotation is

$$d\theta = 2\pi(dx) \text{ radians} \tag{6.6}$$

Each side of the conductor cuts only once through the regions of the north and south magnetic field. That is, the incremental change of the flux linkage for each side of the single-turn armature winding is

$$d\lambda = 2\phi(dx) \tag{6.7}$$

where dx, as before, is the incremental change in the rotation of the rotor. The change in the flux linkage for the entire length of the winding is

$$d\lambda = 4\phi(dx) \text{ for a two-pole machine} \tag{6.8}$$

$$= 2p\phi(dx) \text{ for a } p\text{-pole machine} \tag{6.9}$$

The current through the single winding is determined by the number of parallel paths of the armature winding and is related to the external armature current (I_a) by

$$i = \frac{I_a}{\beta} \tag{6.10}$$

where

β = the number of parallel paths of the armature winding

β = number of machine poles, for *lap-type** armature winding

β = 2, for *wave-type*† armature winding.

From Eqs. (6.5), (6.6), (6.9), and (6.10), we see that the expression for the torque developed by a single conductor is

$$T_i = \frac{p}{\beta\pi}\phi I_a \text{ N} \cdot \text{m/turn} \tag{6.11}$$

For N turns of the armature winding, the total torque developed (T) by the motor is

$$T = \frac{Np}{\beta\pi}\phi I_a \text{ N} \cdot \text{m} \tag{6.12}$$

or

$$T = K\phi I_a \text{ N} \cdot \text{m} \tag{6.13}$$

**Lap-type winding*: The starting and terminating ends of adjacent coils overlap each other's winding and are connected to adjacent commutator segments. The number of parallel paths is equal to the number of poles. This type of winding is used for low-voltage, high-current applications.

†*Wave-type winding*: The coils are connected in two parallel paths regardless of the number of poles. Most dc machines with ratings less than 75 kW are constructed using this type of winding.

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where the constant of proportionality K is given by

$$K = \frac{Np}{\beta\pi} \tag{6.14}$$

Thus, the torque developed by a dc motor is directly proportional to the machine's effective field, to the armature current, to the number of poles, and to the number of armature conductors.

Generator Principles

The operation of generators is based on the induction principle, whereby voltage is induced in a conductor that is rotated through an external magnetic field. The polarity of the induced voltage, as per Lenz's law, is in such a direction as to produce a current whose mmf opposes the external magnetic field. The conductor under consideration is the coil of the rotor, the external magnetic field is provided by the stator coil, and the conductor's rotation is derived from a prime mover such as an induction motor, a steam or hydroturbine, and so forth.

The derivation of the general expression for the voltage generated is as follows. The voltage induced in a single-turn armature winding (Fig. 6-5) is given by the time rate of change of the armature coil's flux linkage. Considering only magnitudes, we have

$$v_g = \frac{d\lambda}{dt} \tag{6.15}$$

where v_g is the voltage generated and λ is the armature coil's flux linkage.

The change in the flux linkage of the armature winding that rotates through dx of a revolution is, as before,

$$d\lambda = 4\phi(dx) \quad \text{for a two-pole machine} \tag{6.16}$$

$$= 2p\phi(dx) \quad \text{for a } p\text{-pole machine} \tag{6.17}$$

The increment of time that the winding requires to travel through dx of a revolution is

$$dt = \frac{2\pi}{\omega}(dx) \text{ seconds} \tag{6.18}$$

where ω is the angular speed of the armature in radians per second. From Eqs. (6.15), (6.17), and (6.18), we obtain

$$v_g = \frac{p}{\pi}\phi\omega \text{ V/turn} \tag{6.19}$$

The total voltage generated in an armature winding depends on the number of coil turns connected in series. The number of coil turns connected in series

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is equal to the number of conductors divided by the number of parallel paths. That is,

$$\text{number of series conductors} = \frac{N}{\beta} \tag{6.20}$$

From Eqs. (6.19) and (6.20), the total voltage generated (V_g) in the armature conductors is

$$V_g = \frac{Np}{\beta\pi}\Phi\omega \tag{6.21}$$

or

$$V_g = K\phi\omega \tag{6.22}$$

where the constant of proportionality K is the same as that given for the torque equation (see Eq. (6.14)).

The voltage generated in a single conductor, the machine's total voltage, and its rectification are further discussed in Section 6.1.6 and are shown in Fig. 6-10.

The voltage generated in the armature conductors is also referred to as the armature voltage V_a, the countervoltage V_c, the back emf V_b, or simply the speed voltage. You must differentiate between the voltage generated in a machine and its terminal voltage. In this text, the voltage generated will be designated as V_g and the terminal voltage as V_t.

From the preceding discussion it is evident that the principles of dc motor and generator operation are the same as those of polyphase ac machines. The essential difference between a motor and a generator is the direction of energy flow. A motor transforms electrical energy into a mechanical form, whereas a generator transforms mechanical energy into an electrical form.

6.1.3 Power Considerations

The electromagnetic power developed (P_d) by a machine is equal to the product of the electromagnetic torque developed (T) times the speed (ω) of its rotor. In mathematical form,

$$P_d = T\omega \tag{6.23}$$

The torque is expressed in N · m, the speed in rad/s, and the power in watts. Applying the power-balance concept for a motor, we obtain

$$\begin{pmatrix}\text{input}\\ \text{electrical power}\end{pmatrix} = \begin{pmatrix}\text{sum of various}\\ \text{winding copper losses}\end{pmatrix} + \begin{pmatrix}\text{power}\\ \text{developed}\end{pmatrix}$$

Using mathematical symbols,

$$V_tI_L = I_f^2R_f + I_a^2R_a + P_d \tag{6.24}$$

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where

V_t and I_L = the motor's supply (terminal) voltage and line current, respectively

$I_f^2 R_f$ = the copper losses of the field winding

$I_a^2 R_a$ = the copper losses of the armature winding

The power developed provides the output power (P_{out}) plus the rotational losses ($P_{\text{r.l.}}$). Mathematically expressed,

$$P_d = P_{\text{r.l.}} + P_{\text{out}} \tag{6.25}$$

To a certain extent, rotational losses depend on the speed of the motor, but throughout this chapter, they will be assumed to remain constant. At nominal operating conditions, the output power of the motor is referred to as the full-load power, rated power, or nameplate power.

The various losses for a 100 kW dc machine and the direction of energy flow are shown in Fig. 6-6.

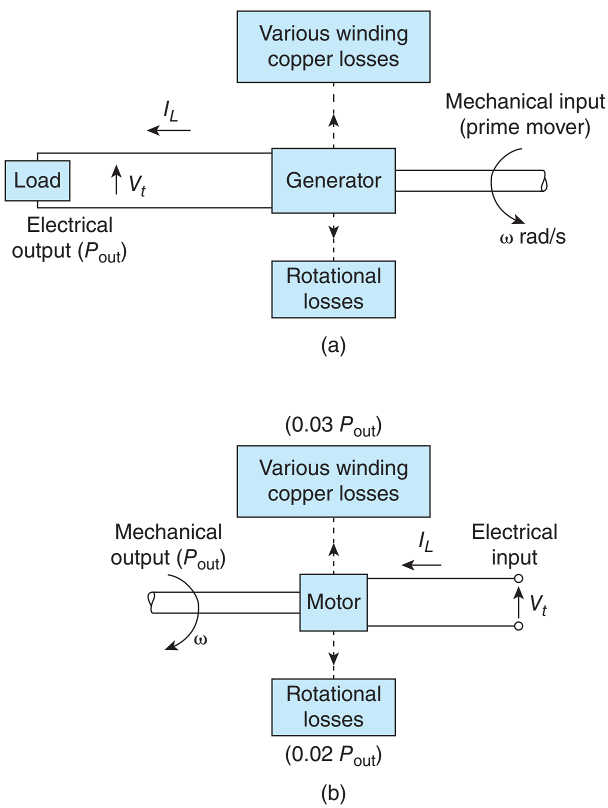

FIG. 6-6 DC machines—direction of power flow and typical losses for a 100 kW machine: **(a)** generator, **(b)** motor.

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EXAMPLE 6-1

A 10 kW, four-pole dc machine is magnetized at a flux level of 0.08 Wb and has a lap-type armature winding with 24 turns. When the motor delivers rated power at 1200 r/min, determine:

a. The voltage induced in the armature windings.
b. The armature current.

SOLUTION

a. The speed of the motor is

$$\omega = 1200 \frac{2\pi}{60} = 125.7 \text{ rad/s}$$

For a lap-type winding, the number of parallel paths is equal to the number of poles. Thus,

$$\beta = 4$$

Substituting the given data into Eq. (6.21), we obtain

$$V_g = \frac{24(4)}{4\pi}(0.08)(125.7) = \underline{\underline{76.8 \text{ V}}}$$

b. From Eq. (6.23), the torque is

$$T = \frac{\text{power}}{\text{speed}} = \frac{10{,}000}{125.7} = 79.6 \text{ N} \cdot \text{m}$$

From Eq. (6.12),

$$I_a = \frac{79.6}{24 \times 4(0.08)}(4\pi) = \underline{\underline{130.21 \text{ A}}}$$

Exercise 6-1

A 5 kW, two-pole dc motor has 200 W of rotational losses and draws an armature current of 30 A when it delivers rated power at 900 r/min. Determine:

a. The voltage generated in the armature windings.
b. The torque developed and the output torque of the motor.

Answer (a) 173.3 V; (b) 55.2 N · m, 53.1 N · m.

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Exercise 6-2 A two-winding transformer has a primary and a secondary winding and two corresponding mmf's. The interaction of these two fields does not produce any torque. What are the reasons for this?

6.1.4 Voltage and Torque Relationships as Functions of Mutual Inductance

This section derives general expressions for the voltage generated and the torque developed in a dc machine as functions of the mutual inductance between the stator and rotor windings.

The voltage induced in the armature of a dc machine with constant field current is given by

$$v_g = -I_f \frac{d}{dt} L_{af(\theta)} \tag{6.26}$$

where $L_{af(\theta)}$ is the mutual inductance between the armature and field windings as a function of the angle of rotation. The mutual inductance is nonsinusoidal in waveform. It has a fundamental term and a higher order harmonics. The negative sign gives the polarity of the induced voltage as per Lenz's law.

Rewriting the relationship of mechanical (θ_m) and electrical (θ) radians (see Eq. (3.15)), we have

$$\theta = \frac{p}{2}\theta_m \tag{6.27}$$

Differentiating with respect to time, we get

$$\frac{d\theta}{dt} = \frac{p}{2}\frac{d\theta_m}{dt} \tag{6.28}$$

From the above,

$$\frac{d\theta}{dt} = \frac{p}{2}\omega \tag{6.29}$$

where p is the number of poles and ω is the speed of the motor in rad/s.

Equation (6.26) can be rewritten as:

$$v_g = -I_f \frac{d\theta}{dt}\frac{dL_{af(\theta)}}{d\theta} \tag{6.30}$$

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From the above and Eq. (6.29), we obtain

$$v_g = -I_f \frac{p}{2} \omega \frac{dL_{af(\theta)}}{d\theta} \tag{6.31}$$

Assuming that the mutual inductance has only a fundamental component, the maximum value of the induced voltage is

$$V_g = \frac{p}{2} \omega I_f L_{\text{afm}} \tag{6.32}$$

where L_{afm} is the maximum value of the fundamental component of the mutual inductance.

The equation of the torque is derived as follows:

$$T = \frac{\text{power}}{\text{speed}}$$

$$= \frac{(\text{voltage induced in the armature})(\text{armature current})}{\text{speed}}$$

Thus,

$$T = \frac{V_g I_a}{\omega} \tag{6.33}$$

From the above and from Eq. (6.32), we obtain

$$T = \frac{p}{2} I_f L_{\text{afm}} I_a \tag{6.34}$$

Comparing Eqs. (6.13) and (6.34) shows that the constant of proportionality of a given dc machine may also be expressed in terms of the mutual inductance between the field and armature windings. That is,

$$K = \frac{p}{2\phi} I_f L_{\text{afm}} \tag{6.35}$$

The maximum value of mutual inductance between the rotor and stator windings can be measured by one of the standard methods (see Chapter 1, Example 1-30), or it can be evaluated indirectly from Eq. (6.32).

The voltage induced in the field windings due to armature current is negligible because the structure of the machine drastically reduces the magnetic coupling of the stator windings to the flux of the armature current.

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Exercise 6-3

A 5 kW, 1200 r/min, four-pole dc machine has an armature current of 20 A and a field current of 2 A. Determine:

a) The amplitude of the fundamental component of the mutual inductance between the stator and rotor windings.

b) The voltage generated.

Answer (a) 0.497 H; (b) 250 V

6.1.5 Magnetic System, Flux Distribution, and Armature Reaction

This section discusses the dc machine's magnetic system, flux distribution, and the interaction of the stator and rotor fields, commonly known as armature reaction.

Magnetic System and Flux Distribution

Figure 6-7 shows an elementary magnetic system for a two-pole dc machine. The flux produced by the mmf of the field winding completes its loop by passing through the air gaps, stator, and rotor structures.

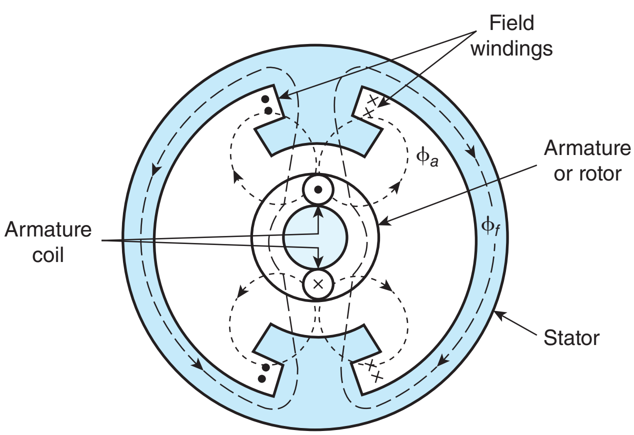

FIG. 6-7 An elementary magnetic system for a two-pole dc machine.

Applying Ohm's law for magnetic circuits, we have

$$\phi_f = \frac{N_f}{\mathcal{R}_f} I_f \tag{6.36}$$

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where the subscript f indicates field parameters. The reluctance ($\mathcal{R}_f$) represents the opposition presented to the field flux by the intrinsic properties of the magnetic circuit components. This reluctance is determined by the sum of the magnetic resistances of the rotor, stator, and air-gap paths, through which the field flux completes its loop.

In general,

$$\begin{aligned}\mathcal{R} &= \frac{l}{\mu A}\\ &= \frac{\text{length of closed magnetic path}}{(\text{permeability}) \times (\text{cross-section area perpendicular to the flux})}\end{aligned} \tag{6.37}$$

Taking into consideration the three distinct parts of the field's magnetic circuit, we have

$$\mathcal{R}_f = \frac{l_r}{\mu_r A_r} + \frac{l_s}{\mu_s A_s} + \frac{l_g}{\mu_g A_g} \tag{6.38}$$

where the subscripts r, s, and g represent, respectively, the usual parameters of rotor, stator, and air gaps through which the field flux closes its magnetic circuit.

For operations along the linear section of the magnetic characteristic, the permeability is constant. Thus from Eqs. (6.36) and (6.38), we obtain

$$\phi_f = K_f I_f \tag{6.39}$$

where K_f is a constant of proportionality that is proportional to the number of field winding turns and inversely proportional to the reluctance of the field's magnetic circuit.

Similarly, the flux of the armature coils (ϕ_a) is determined by

$$\phi_a = K_a I_a \tag{6.40}$$

where I_a is the armature current and K_a is a constant of proportionality which, as before, depends on the reluctance of the armature's magnetic circuit and on the number of turns of the armature winding.

Equations (6.39) and (6.40) are *not* applicable for machine operations along the nonlinear part of the magnetization characteristic.

Flux-Density Distribution

Figure 6-8 shows a simplified version of the flux-density distribution of a two-pole dc machine. The armature (B_a) and field (B_f) flux densities combine to produce the resultant (B_r) or effective flux density.

The field flux density under the north magnetic pole is designated as positive, while that under the south magnetic pole is negative. The relative polarity becomes obvious when one considers that in an N–S magnetic system, the lines of force leave the north pole and enter the south pole.

The distribution of the armature flux density is of lower amplitude because of the larger reluctance path through which the armature flux completes its magnetic

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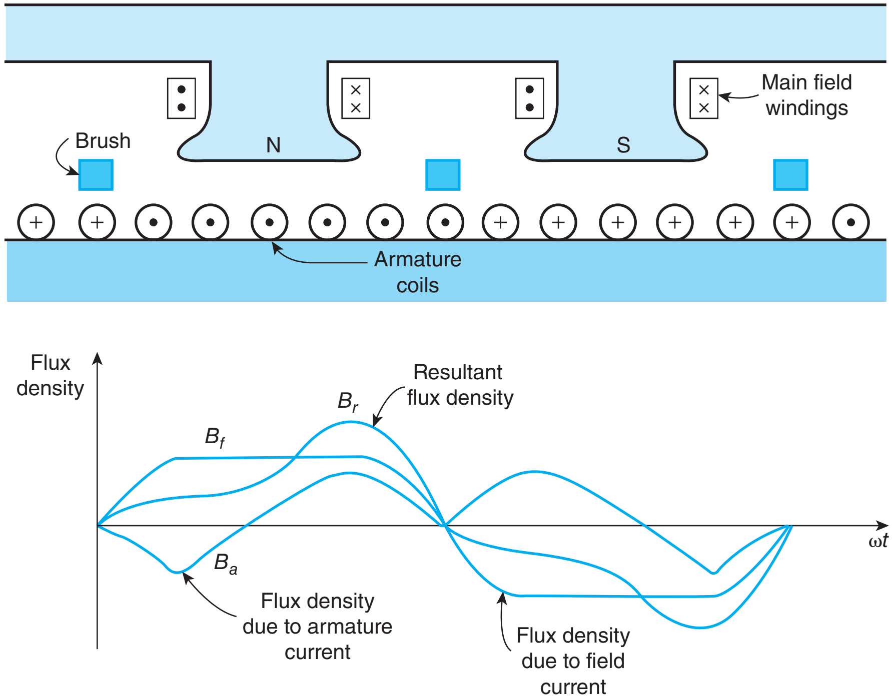

FIG. 6-8 Simplified view of a two-pole machine and the corresponding flux-density distributions. (Interpoles are not shown.)

circuit. Notice also that for one complete revolution of the rotor, each side of the armature coil goes through a complete magnetic circuit. As a result, the armature coil will appear to travel two cycles for each magnetic field cycle.

The flux-density distribution is zero under the stationary brushes because the stationary brushes are physically located at a point between the north and south magnetic poles where the field is zero.

The Concept of Armature Reaction

The flux of the armature current effectively *opposes* the flux of the field current and *distorts* the waveform of the field's flux-density space distribution. These two adverse effects of the armature flux are referred to as armature reaction. The qualitative analysis of the armature reaction is described next, while its quantitative analysis is given in Section 6.1.9.

Opposition to the Field

Depending on the rotor position, the flux of the armature *aids* the flux of the field on one side of the pole and *opposes* it on the other side. Aiding the field in one pole partly saturates the magnetic material, and thus the effective field is reduced.

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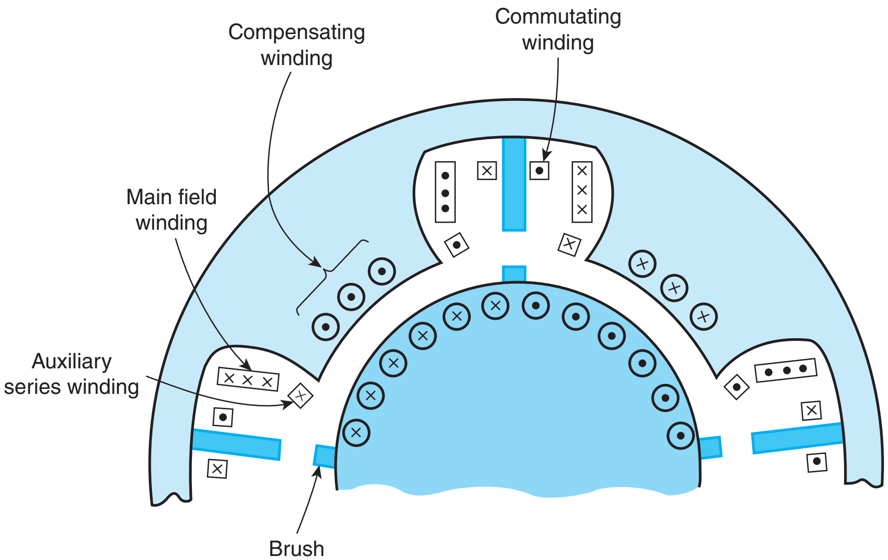

FIG. 6-9 DC machines. This figure illustrates the physical location and relative polarity of compensating and commutating windings.

The overall effect of the armature mmf is, then, to distort and/or to oppose the flux of the field. This opposition, or armature reaction (AR), is usually less than about 5% of the field's flux at no-load.

With constant field current and rated armature current, an armature reaction of 4% means that

$$\phi_2 = 0.96\phi_1$$

where ϕ_1 is the machine's flux at negligible armature current ($I_{a_1} = 0$) and ϕ_2 is its effective flux at rated armature current (I_{a_2}). In other words, at full-load, the effective magnetic field is 96% of the field at no-load.

The cross-magnetizing effect of the armature current may be minimized by placing, in longitudinal slots in the pole faces, a *compensating winding* (see Fig. 6-9). This winding is connected in series with the armature and hence carries the same current, but in the direction opposite to that of the adjacent armature conductors.

Distortion of the Field's Flux Distribution

As shown in Fig. 6-8, the flux of the armature coil distorts the field in the interpole region, particularly where the commutator segment rides under the positive or the negative polarity brush. This results in a nonlinear variation of the armature current, which can lead to severe sparking. In fact, one of the essential limitations of dc machines is the arcing and possible flashover that may accompany the reversal of the armature current, commonly known as commutation.

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(Commutation will be explained more fully in the next section.) To cancel this adverse effect, a coil—the so-called *commutating winding*—is wound on the narrow poles of the interpole region and connected in series with the armature circuit. When properly designed, a commutating winding can minimize the distorting effect of the armature mmf on the field flux distribution.

6.1.6 Commutation

The simplified schematic of Fig. 6-10(a) shows that during one complete revolution of the shaft, each side of the armature conductor is rotated through two opposite magnetic fields (N and S). Thus the voltage, or the current, would be positive or negative, or approximately sinusoidal in waveform, as shown in Fig. 6-10(b).

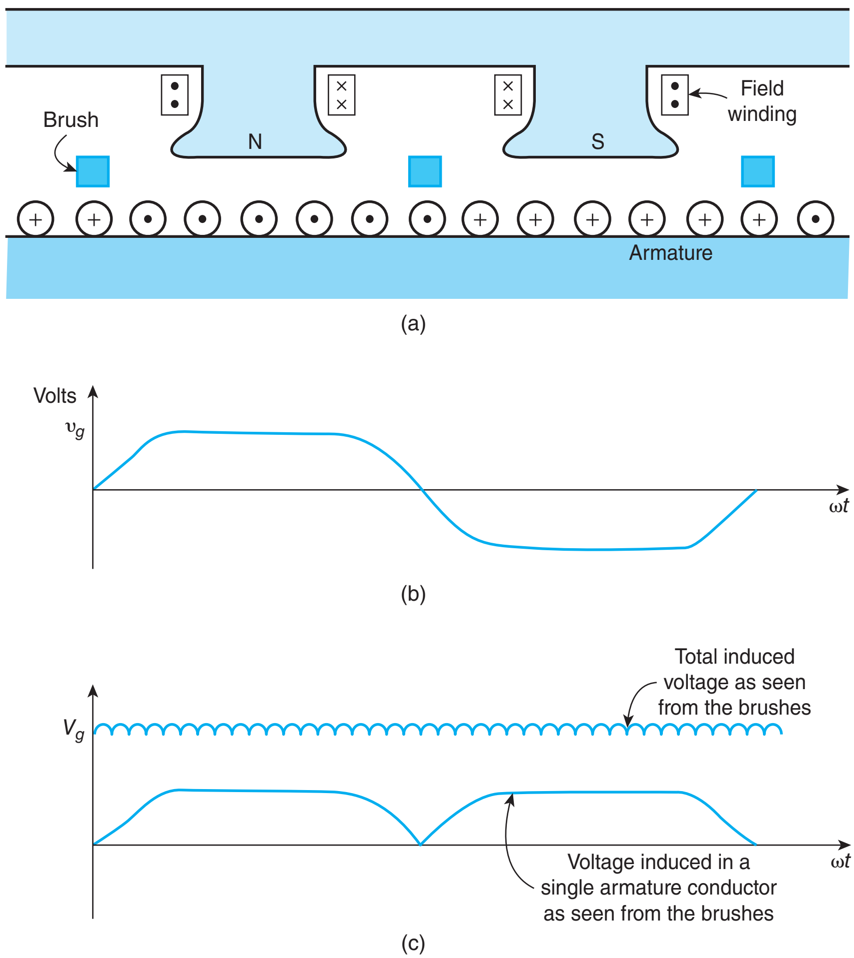

FIG. 6-10 **(a)** Simplified view of a two-pole machine; **(b)** voltage induced in a single armature conductor; **(c)** effect of the brushes on the induced voltage.

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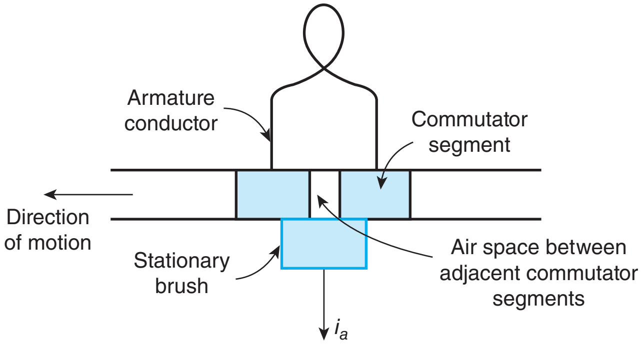

FIG. 6-11 Commutation, showing the location of brush and commutator segments when an armature conductor is shorted.

Properly reversing the end connections of the conductor causes a unidirectional or dc current to flow in the machine's external circuit. The current is reversed through the brushes, whose polarity is fixed by their physical location relative to the north and south poles of the field winding. The reversal of the armature current during half the period of its cycle is called *commutation*.

As the location of the brushes indicates, the commutation starts when the conductors are at the region of the weakest magnetic field (see Fig. 6-10(c)). Ideal or sparkles commutation occurs when the armature current is made to vary linearly during the commutation period. This occurs when the resistance of the shorting brush(es) is the predominant characteristic of the shorted armature conductor.

The instant the conductor leaves the north pole of the magnetic field and is about to enter the south pole, the brush, as shown in Fig. 6-11, short-circuits the conductor. As a result, sparking may take place. The degree of the arcing will depend on the magnitude of the armature current, on the voltage induced in the shorted conductor, and on the resistance of the short-circuited path. Arcing is also increased by the distortion of the air-gap field that results from the armature reaction. The distortion of the air-gap field, as already discussed, can be completely neutralized by properly designing the commutating winding.

Excessive sparking may result in the breakdown of the air space between the commutator segments, and thus a flashover may engulf the entire periphery of the commutator. The flashover, as seen from the terminals of the motor, constitutes a short circuit to the external voltage source.

Mathematical Considerations

If one neglects the coil resistance, the voltage induced in an armature coil (V_{coil}) depends on its self-inductance, its mutual inductance, and the speed of the rotor. Mathematically,

$$V_{\text{coil}} = K\phi\omega - \frac{d\lambda_a}{dt} - \frac{d}{dt}(i(t)L(t)) \qquad \textbf{(6.41)}$$

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where:

$K\phi\omega$ = the speed voltage or the voltage generated due to the induction principle. During the commutation period, this voltage changes because of variations in the magnitude and direction of the air-gap flux density. Its magnitude can be controlled by properly modifying the air-gap field distribution in the commutating region.

$\frac{d}{dt}(\lambda_a) = L_a \frac{di_{a(t)}}{dt}$ = the voltage drop caused by the coil's self-inductance and the rate of change of its current. During the commutation period, this component of the voltage changes, because the current changes in magnitude and direction. By definition, this voltage is also given by the rate of change of the flux linkage of the coil undergoing commutation.

$\frac{d}{dt}(i(t)L(t))$ = the voltage drop due to the conductor's mutual ($L(t)$) inductance with respect to the adjacent conductors that carry an instantaneous current of $i(t)$ amperes. Here, only one term is considered, but it is obvious that almost all coils are mutually coupled to the conductor that is undergoing commutation.

Before commutation takes place, these voltages are balanced by the external voltage. During commutation, however, the coil is short-circuited, and the sum of these voltages must be equal to zero. The generation or development of sparks results from nature's attempt to set up a voltage that satisfies KVL.

6.1.7 Equivalent Circuits and External Machine Characteristics

The equivalent circuit of a dc machine explicitly shows how the field winding is excited in relation to the armature winding. As such, an equivalent circuit reveals the type of machine (series, shunt, separately excited, etc.) it represents. An equivalent circuit may include the actual values of the winding impedances, so that it can be used for transient and steady-state analyses. In this section, however, only steady-state analyses are discussed. Consequently, the equivalent circuits include only the resistance of the armature (R_a) and field windings (R_f).

The resistance of the armature winding is relatively small, normally a fraction of an ohm. It includes the resistances of the armature conductors and the resistances of the compensating and commutating windings. The effects of the contact between the brush and the commutator segments is represented by a constant voltage drop (about 1 V).

The resistance of the field winding, when connected in series with the armature winding, is very small. When connected in parallel to the supply voltage, however, it is several hundred times larger than the resistance of the armature windings.

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The external characteristics of dc machines can easily be derived from the basic motor-generator relationships and from the equations that can be obtained from the machine's equivalent circuit.

Shunt dc Motors

The equivalent circuit of a dc shunt motor is shown in Fig. 6-12(a). The voltage V_t represents the motor's dc supply or terminal voltage, and I_a represents the current of the armature windings. The armature current is determined by the supply voltage, the driven load, and the field current. The resistance of the field winding ($R_f > 100\ \Omega$) is much greater than the resistance of the armature coil ($R_a < 0.2\ \Omega$), and under normal operating conditions, the field current is much smaller than the armature current.

Ordinarily, a dc shunt motor is equipped with a "starting box" that houses an external resistor whose value can be decreased in several steps. The purpose of the external resistor is to limit the armature current during starting (see Section 6.1.8).

When stopping a dc shunt motor, the armature circuit should be opened either before or at the same time as the field circuit. If the field current is interrupted while the armature circuit is energized, severe sparking on the commutator will result, and the motor's speed will increase to dangerously high levels.

For purposes of speed control, the motor may be equipped with variable external resistors in the armature (R_{a_x}) and field (R_{f_x}) circuits. The motor's external characteristics are derived as follows.

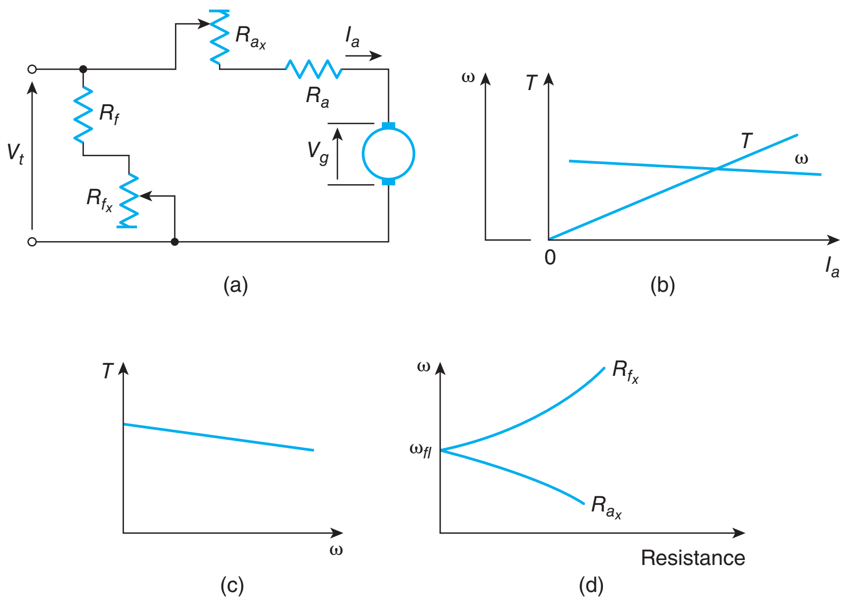

FIG. 6-12 Shunt dc motors: **(a)** equivalent circuit including the rheostat's resistances, **(b)** torque and speed versus armature current, **(c)** torque versus speed, **(d)** speed versus rheostat resistances.

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Torque versus Armature Current

The basic motor-torque equation for a constant field current and negligible armature reaction becomes

$$T = K_1 I_a \tag{6.42}$$

That is, the torque is directly proportional to the armature current. The torque-armature current characteristic is shown in Fig. 6-12(b).

Speed versus Armature Current

From the basic generator equation and KVL in the equivalent circuit ($R_{a_x} = 0$), we have

$$V_g = K\phi\omega \tag{6.43}$$

$$V_t = V_g + I_a R_a \tag{6.44}$$

From the above,

$$\omega = \frac{V_t}{K\phi} - \frac{R_a}{K\phi} I_a \tag{6.45}$$

For constant field current, the coefficient of I_a is small, and thus the speed of the motor does not appreciably change with moderate variations in the armature current. (See Fig. 6-12(b).)

Torque versus Speed

From Eqs. (6.43) and (6.44), we obtain

$$V_t = I_a R_a + K\phi\omega \tag{6.46}$$

Also, from the basic torque equation, we have

$$T = K\phi I_a \tag{6.47}$$

From the last two equations, we obtain

$$T = \frac{K\phi}{R_a} V_t - \frac{(K\phi)^2}{R_a}\omega \tag{6.48}$$

For a constant field current, the last equation is of the intercept-slope form, and because the slope is small, moderate variations in the speed do not appreciably change the torque. The torque-speed characteristic of the shunt motor is shown in Fig. 6-12(c).

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Speed versus External Resistors

The speed of the shunt motor can be controlled through the external armature and field resistors.

Effects of Armature Rheostat (R_{ax})

Applying KVL in the equivalent circuit, we obtain

$$V_t = V_g + I_a(R_a + R_{ax}) \tag{6.49}$$

From Eqs. (6.43) and (6.49), we get

$$\omega = \frac{V_t - I_a R_a}{K\phi} - \frac{I_a}{K\phi} R_{ax} \tag{6.50}$$

For constant field and armature currents, the larger the external armature resistance, the smaller will be the motor's speed.

Equation (6.50) makes it evident that the external armature resistance can control the speed of the motor from zero up to its full-load value. This method of speed control is accompanied by high copper losses ($I_a^2 R_{ax}$). For this reason, almost all modern speed-control apparatuses use solid-state devices to reduce the terminal voltage and thus to control the speed of the motor. The speed of a shunt motor as a function of the external armature resistance is shown in Fig. 6-12(d).

Effects of the Field Rheostat (R_{fx})

From the speed-voltage equation and Ohm's law, across the field winding we have

$$V_g = K_1 I_f \omega \tag{6.51}$$

$$I_f = \frac{V_t}{R_f + R_{f_x}} \tag{6.52}$$

From Eqs. (6.44), (6.51), and (6.52), we obtain

$$\omega = \frac{(V_t - I_a R_a)(R_f + R_{f_x})}{K_1 V_t} \tag{6.53}$$

Since $V_t \gg I_a R_a$, the last expression becomes

$$\omega \approx \frac{R_f + R_{f_x}}{K_1} \tag{6.54}$$

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Equation (6.54) shows that by increasing the external field resistance, the speed of the motor is also increased. The speed of the motor as a function of the field rheostat resistor is shown in Fig. 6-12(d).

The maximum attainable speed is limited by sparking in the brushes or by mechanical considerations of the centrifugal forces.

Example 6-2

A 10 kW, 220 V shunt dc motor has field and armature resistances of 110 ohms and 0.20 ohms, respectively. At no-load, the motor runs at 1200 r/min and has an armature current of 5.0 A. Determine the speed and electromagnetic torque when the motor draws 50 A from the supply. Assume an armature reaction that is:

a. Negligible
b. 4%.

Solution

a. The parameters marked with the subscript 1 will correspond to the no-load condition, and those marked with subscript 2 will correspond to the load condition of 50 A. Applying KVL to Fig. 6-13, we obtain

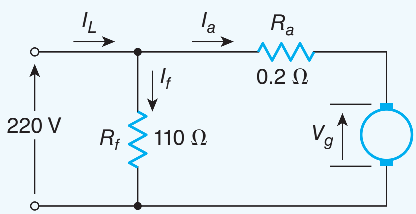

FIG. 6-13

$$V_{g_1} = V_t - I_{a_1} R_a$$
$$= 220 - 0.2(5) = 219 \text{ V}$$

When the motor draws 50 A, the armature current will be

$$I_{a_2} = I_L - I_f = 50 - \frac{220}{110} = 48 \text{ A}$$

and

$$V_{g_2} = 220 - 0.2(48) = 210.4 \text{ V}$$

For negligible armature reaction and constant field current,

$$\phi_1 = \phi_2$$

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From the generator principle, we have

$$\frac{V_{g_2}}{V_{g_1}} = \frac{\phi_2}{\phi_1} = \frac{n_2}{n_1}$$

From the above,

$$n_2 = 1200\,\frac{210.4}{219} = \underline{\underline{1152.88 \text{ r/min}}}$$

Besides giving the rotational losses, the no-load data can also be used to establish the motor constant, as follows:

$$V_{g_1} = K\phi\omega_1$$

For constant field,

$$V_{g_1} = K_1\omega_1$$

Thus,

$$K_1 = \frac{219}{1200\,\frac{2\pi}{60}} = 1.74 \text{ V/rad/s}$$

The electromagnetic torque developed is

$$\begin{aligned} T &= K\phi I_a \\ &= K_1 I_a \\ &= 1.74 \times 48 = \underline{\underline{83.65 \text{ N} \cdot \text{m}}} \end{aligned}$$

Alternatively,

$$T = \frac{V_{g_2} I_a}{\omega} = \frac{210.4\,(40)}{1152.88\,\frac{2\pi}{60}} = \underline{\underline{83.65 \text{ N} \cdot \text{m}}}$$

b. For an armature reaction of 4%, we have

$$\phi_2 = 0.96\phi_1$$

From the generator principle, we obtain

$$n_2 = \frac{V_{g_2}\phi_1}{V_{g_1}\phi_2}\,n_1$$

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Substituting the known parameters, we obtain

$$n_2 = 1200\,\frac{210.4}{219}\left(\frac{1}{0.96}\right) = \underline{\underline{1200.91\ \text{r/min}}}$$

The torque is found as follows:

$$\begin{aligned} T &= K\phi_2 I_{a_2} \\ &= 0.96(K\phi_1 I_{a_2}) = 0.96(1.74 \times 48) \\ &= \underline{\underline{80.31\ \text{N}\cdot\text{m}}} \end{aligned}$$

For purposes of comparison, the results are summarized in Table 6-1. Clearly, the armature reaction increases the speed and reduces the torque of the motor by a percentage about equal to that of the armature reaction.

TABLE 6-1 Summary of results of Example 6-2

Assumed Armature Reaction	Armature Current	Speed r/min	Torque Developed (N · m)
0	48	1152.88	83.65
4%	48	1200.91	80.31

Series dc Motors

In a series dc motor, the field winding is connected in series with the armature winding (see Fig. 6-14(a)). For speed control, a variable resistor (R_{f_x}) is often connected in parallel to the field winding. The motor's external characteristics are derived as follows.

Torque versus Armature Current

In a series dc motor *without* an external field resistor, the field current is equal to the armature current. Hence, for operations along the linear portion of the magnetic characteristic, and in the absence of armature reaction, the machine's flux is directly proportional to the armature current. Rewriting the basic torque equations, we have

$$T = K\phi I_a \tag{6.55}$$

or

$$T = K_1 I_a^2 \tag{6.56}$$

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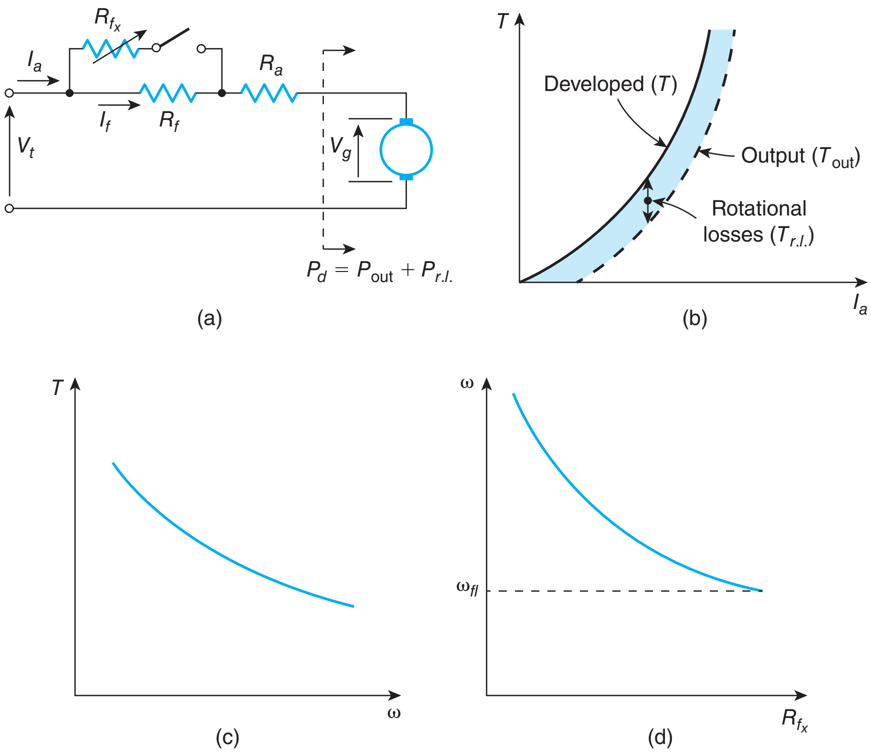

FIG. 6-14 Series dc motors: **(a)** equivalent circuit, **(b)** torque versus armature current, **(c)** torque versus speed, **(d)** speed versus external field resistance.

In other words, the torque developed by a series dc motor is directly proportional to the square of the armature current. The torque-versus-armature-current characteristic is shown in Fig. 6-14(b). The actual curve, however, is displaced downward because of the machine's rotational losses.

Torque versus Speed

Applying KVL in the circuit of Fig. 6-14(a), we obtain

$$V_t = V_g + I_a(R_a + R_f) \tag{6.57}$$

Substituting for voltage generated its equivalent equation, we obtain

$$V_t = K_1 I_a \omega + I_a(R_a + R_f) \tag{6.58}$$

From Eqs. (6.56) and (6.58), we find

$$T = K_1 \left(\frac{V_t}{K_1\omega + R_a + R_f} \right)^2 \tag{6.59}$$

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Because the resistance of the armature and series field windings is relatively small, the equation becomes

$$T \approx \frac{V_t^2}{K_1\omega^2} \tag{6.60}$$

The torque of a series motor, then, is inversely proportional to the square of the speed. As a result, series dc motors are suitable for mechanical loads that require high torque at low speeds and lower torque at higher speeds, so they are used in trains, trolleys, and the like.

The torque-speed characteristic is shown in Fig. 6-14(c).

Speed versus External Field Resistance (R_{f_x})

In practice, the speed of a series motor is controlled through an external field resistor (R_{f_x}), which is connected in parallel to the field winding. The governing equation that describes the effect of the external field resistance on the speed of the motor is derived as follows.

By assuming operation along the linear part of the magnetization characteristic, the basic expression for the armature voltage becomes

$$V_g = K_1 I_f \omega \tag{6.61}$$

Neglecting the voltage drop across the small armature and field resistors, the voltage generated is equal to the terminal voltage; thus,

$$V_t \approx K_1 I_f \omega \tag{6.62}$$

Considering the field resistors, from the current-divider concept, we obtain

$$I_f = I_a \frac{R_{f_x}}{R_f + R_{f_x}} \tag{6.63}$$

From Eqs. (6.62) and (6.63), we get

$$\omega = \frac{V_t}{K_1 I_a R_{f_x}}(R_f + R_{f_x}) \tag{6.64}$$

or

$$\omega = \frac{V_t}{K_1 I_a}\left(1 + \frac{R_f}{R_{f_x}}\right) \tag{6.65}$$

For a given armature current, then, the larger the external field resistance is, the smaller the speed of the motor will be. The corresponding motor characteristic is shown in Fig. 6-14(d).

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EXAMPLE 6-3

A 300 V, 60 A, 1200 r/min series dc motor has an armature and a series field resistance, each of which is 0.20 ohm. When a 0.10 ohm resistance is connected in parallel to the field winding, the motor's torque is doubled. Assuming negligible armature reaction and rotational losses, determine the motor's:

a. Armature current.
b. Speed.
c. Efficiency.

SOLUTION

The given nameplate data establishes the constant of the motor. Using the equivalent circuit of Fig. 6-15(a), at 1200 r/min, we have

$$V_{g_1} = V_t - I_{a_1}R_{t_1}$$
$$= 300 - 60(0.2 + 0.2) = 276 \text{ V}$$

As before, subscript 1 designates the motor's parameters at rated conditions, and subscript 2 the parameters that correspond to the new motor's operating condition.

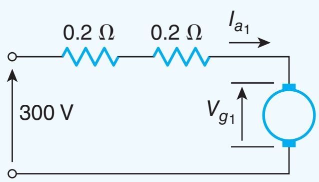

FIG. 6-15(a)

From the generated voltage equation, we also have

$$V_{g_1} = K_1 I_{f_1} \omega$$

Solving for the constant of proportionality, we obtain

$$K_1 = \frac{276}{(60)1200\left(\frac{2\pi}{60}\right)}$$
$$= 0.0366 \text{ V/rad/s/field A}$$

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a. The current through the 0.10 ohm resistor that is connected in parallel with the field winding (see Fig. 6-15(b)) does *not* contribute to the effective flux within the machine because this resistance is physically connected outside the machine's structure.

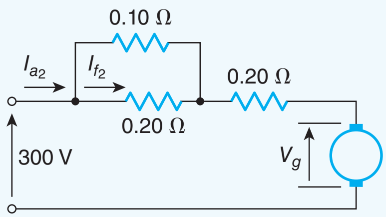

FIG. 6-15(b)

Using the current-divider concept, we obtain

$$I_{f_2} = I_{a_2}\left(\frac{0.1}{0.1 + 0.2}\right) = \frac{I_{a_2}}{3}$$

From the statement of the problem,

$$T_2 = 2T_1$$

or

$$K_1 I_{f_2} I_{a_2} = 2K_1 I_{f_1} I_{a_1}$$

From the above,

$$I_{a_2}^2\left(\frac{1}{3}\right) = 2I_{a_1}^2$$

Substituting and solving for the unknown, we obtain

$$I_{a_2} = 60\sqrt{6} = \underline{\underline{146.97\text{ A}}}$$

b. The speed of the motor is determined as follows. From KVL, we have

$$\begin{aligned} V_{g_2} &= 300 - 146.97(0.2 + 0.10 \,//\, 0.20) \\ &= 260.81\text{ V} \end{aligned}$$

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and

$$\omega_2 = \frac{V_{g_2}}{K_1 I_{f_2}}$$

$$= \frac{260.81}{0.0366\left(\dfrac{146.97}{3}\right)}$$

$$= 145.46 \text{ rad/s}$$

and

$$n_2 = \underline{\underline{1389 \text{ r/min}}}$$

c. The efficiency of the motor is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}}$$

$$= 1 - \frac{I_a^2(R_a + R_f)}{V_t I_a}$$

$$= 1 - \frac{(146.97)^2(0.2 + 0.10 \,/\!/\, 0.20)}{300\,(146.97)} = 0.87$$

Alternatively,

$$\eta = \frac{V_{g_2} I_{a_2}}{V_t I_{a_2}} = \frac{260.81}{300}$$

$$= \underline{\underline{0.87}}$$

For purposes of comparison, the results are summarized in Table 6-2.

TABLE 6-2 Summary of results of Example 6-3

Load Torque	*External Field Resistor (ohms)*	*Speed (r/min)*	*Line Current (A)*
T_1	∞	1200	60
$2T_1$	0.1	1389	146.97

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Exercise 6-4

For a series dc motor, show that the speed as a function of armature current is given by

$$\omega = \frac{V_t}{K_1 I_a} - \frac{R_a + R_f}{K_1}$$

where K_1 is the motor's constant in V/A/rad/s.

Separately Excited dc Motors

In a separately excited dc motor, the field coil is supplied from a different voltage source than that of the armature coil, as shown in Fig. 6-16(a). The field circuit normally incorporates a rheostat through which the field current, and thus the motor's characteristics, can be externally controlled. This motor is suitable primarily for two types of loads: those that require constant torque for speed variations up to full-load speed and those whose power requirements are constant for speed variations above nominal speed.

The power-versus-speed and torque-versus-speed characteristics of such loads are shown in Fig. 6-16(b) and 6-16(c), respectively. These diagrams also identify the method of a motor's speed control. The terminal voltage (V_t) control is used for loads that require constant torque for speed variations *up to* full-load, while the field current (I_f) control is used for constant power requirements for speed variations *above* the full-load speed. Both methods of speed control aim to

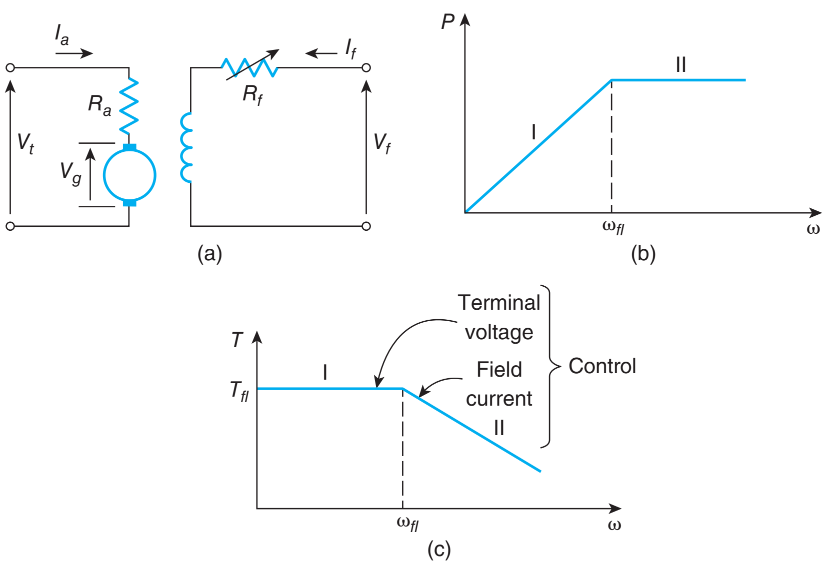

FIG. 6-16 Separately excited dc motor: **(a)** equivalent circuit, **(b)** power-speed characteristic of a load, **(c)** speed-torque characteristic of the load in (b).

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supply the requirements of the mechanical load while not exceeding the nominal capability or rating of the machine.

Each mode of speed control is justified by considering the following basic torque and armature–current relationships:

$$T = K\phi I_a \tag{6.66}$$

$$I_a = \frac{V_t - K\phi\omega}{R_a} \tag{6.67}$$

Terminal Voltage Control

From Eq. (6.66) for constant torque and armature current, the field current must also be constant. Then the armature current, as can be seen from Eq. (6.67), will be kept at its rated value, provided that the terminal voltage increases according to the increase in the speed.

Field Control

For loads that require constant power at speeds higher than rated value, the motor's terminal voltage is kept constant, and the increase of the speed is achieved by reducing the field current. The reduction of the field current for constant armature current results in a reduction of the torque and a corresponding increase in the speed. In practice, the speed control through the field current or through the terminal voltage is accomplished by using the solid-state speed-control apparatus (see Section 6.2).

EXAMPLE 6-4

A 220 V, 4 kW, 22 A, 1260 r/min, separately excited dc motor is driving a fan whose torque is proportional to the square of the motor's speed. The resistance of the armature winding is 0.50 ohm. Neglecting armature reaction and assuming that the excitation remains constant:

a. Determine the terminal voltage required to lower the speed of the fan by 100 r/min.
b. Sketch the torque-speed characteristics of the motor and the fan. Identify the operating points.

SOLUTION

a. The generated voltage at 1260 r/min is found by applying KVL in the armature circuit of Fig. 6-17(a).

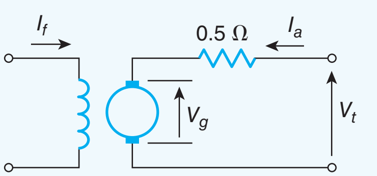

FIG. 6-17(a)

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$$V_{g_1} = V_t - I_a R_a$$
$$= 220 - 22(0.5) = 209 \text{ V}$$

From the generator principle, for constant field current we have

$$K_1 = \frac{V_{g_1}}{\omega} = \frac{209}{1260 \dfrac{2\pi}{60}}$$
$$= 1.58 \text{ V/rad/s}$$

From the motor principle and the given torque-speed relationship, we obtain

$$\frac{T_1}{T_2} = \frac{I_{a_1}}{I_{a_2}} = \left(\frac{\omega_1}{\omega_2}\right)^2$$

Solving for I_{a_2}, and substituting the known parameters, we obtain

$$I_{a_2} = 22\left(\frac{1160}{1260}\right)^2 = 18.65 \text{ A}$$

The new generated voltage is

$$V_{g_2} = K_1\omega = 1.58\left(\frac{1160}{60}\right)2\pi = 192.41 \text{ V}$$

From KVL, the required terminal voltage is

$$V_t = 18.65(0.5) + 192.41 = 201.74 \text{ V}$$

b. The torque-speed characteristic of the fan is as shown in Fig. 6-17(b). The torque of the motor at a given speed must be equal to the torque requirement of the fan at the same speed.

The general expression for the torque-speed characteristic of the motor as a function of the terminal voltage is derived as follows:

$$T = K\phi I_a = K_1 I_a = K_1\left(\frac{V_t - V_g}{R_a}\right)$$
$$= K_1 \frac{V_t}{R_a} - \frac{K_1}{R_a}(K\phi\omega)$$
$$= K_2 V_t - K_3\omega$$

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The constants of proportionality, K_2 and K_3, have been introduced in order to shorten the algebra. The last expression is sketched in Fig. 6-17(b).

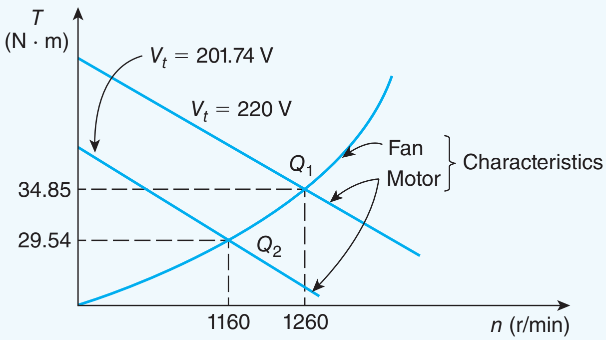

FIG. 6-17(b)

The torque at the given speeds is

$$T_1 = 1.58(22) = 34.85 \text{ N} \cdot \text{m}$$

$$T_2 = 1.58(18.65) = 29.54 \text{ N} \cdot \text{m}$$

The points Q_1 and Q_2, located at the intersection of the motor-load characteristics, represent the operating points.

Exercise 6-5

Redo Example 6-4 by assuming that the load's speed requirement is 1360 r/min instead of 1160 r/min. Will the motor overheat?

Answer 238.41 V

Compound dc Machines

Shunt dc machines may be equipped with an *auxiliary field* winding. The function of this winding is to modify the machine's characteristics to make it more suitable for a particular load requirement.

The auxiliary and shunt field windings are wound around the main field poles. The auxiliary winding has very few turns. It carries either line or armature current, depending on its connection.

When equipped with an auxiliary field winding, shunt dc machines are called *compound* dc machines. Compound machines are further designated as cumulatively

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compounded or differentially compounded, depending on whether the auxiliary winding aids or opposes the main field winding.

When the auxiliary winding is connected *between* the terminal voltage and the junction of the armature and field circuits, the shunt machine is said to be connected in "short-shunt" compounded form. When the auxiliary field winding is connected *in series* with the armature resistance, the machine is said to be connected in "long-shunt" compounded form.

The equivalent circuits of the various compound dc motors are shown in Fig. 6-18. Normally, the resistances of the commutating and the compensating windings (that are part of most dc machines) are not shown separately but are included in the armature resistance.

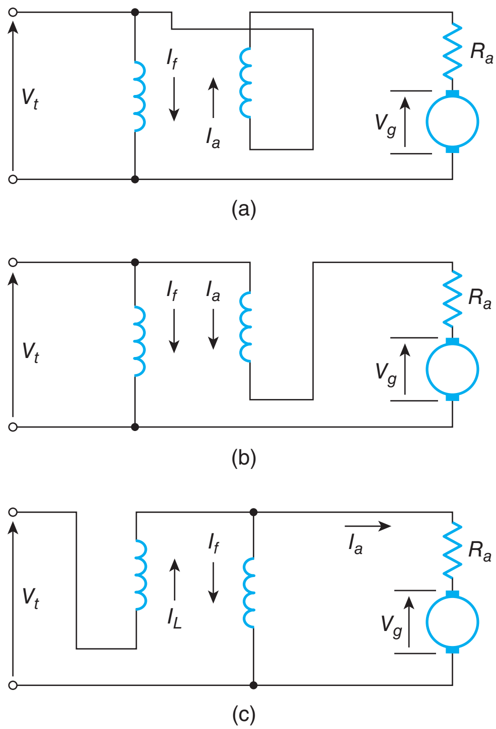

FIG. 6-18 Equivalent circuits of compound dc motors: **(a)** long-shunt, differentially compounded, **(b)** long-shunt, cumulatively compounded, **(c)** short-shunt, differentially compounded.

As shown in Fig. 6-19, the torque-speed characteristic of a compound dc motor is essentially between that of the shunt and the series motor. Actually, a variation of the motor characteristic can be obtained by properly sizing and connecting the auxiliary winding.

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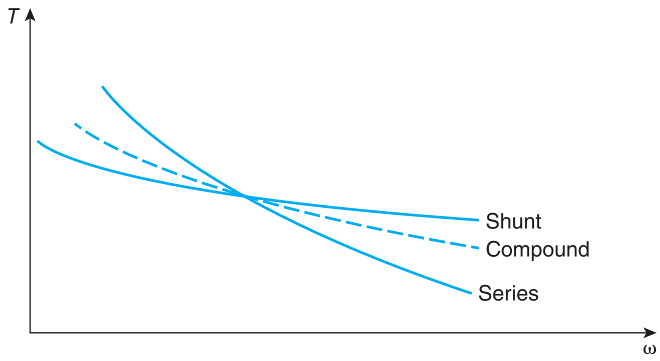

FIG. 6-19 Torque-speed characteristics.

Magnetic Circuit

Figure 6-20 shows the equivalent magnetic circuit per pole of a cumulatively compounded, long-shunt motor. The effective magnetic flux within the machine is essentially produced by the mmf ($N_f I_f$) of the main field.

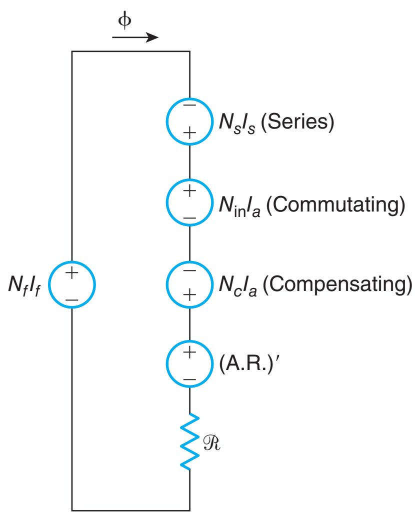

FIG. 6-20 Magnetic equivalent circuit of a long-shunt, cumulatively compounded motor.

The mmf contribution of the *auxiliary series* winding ($N_s I_s$) is shown as aiding the mmf of the main winding. However, the auxiliary series winding should be of opposite polarity if the machine is differentially compounded. In the long-shunt connection, the current through the auxiliary winding is equal to the armature current. In the short-shunt connection, the current through this winding is equal to the line current.

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The armature reaction (A.R.)′, in equivalent magnetic volts, is of a polarity opposite to that of the main field. The compensating winding is designed in such a way that its mmf ($N_c I_a$) is equal and opposite to the armature reaction.

The mmf of the *commutating* or *interpole* winding ($N_{in} I_a$) has a qualititative effect on the flux density of the field winding, for it is specifically designed to minimize the distortion of the armature mmf on the main field flux density at the interpole region.

The effective reluctance $\mathscr{R}$ represents the sum of the reluctances of the air gaps, of the stator path, and of the path through the rotor of the machine. At relatively low flux levels, the reluctance of the air gap predominates. At higher flux densities, the reluctance of the constituent magnetic materials, because of saturation, is the controlling factor.

6.1.8 Starting

The equivalent circuit of a shunt dc motor is shown in Fig. 6-21(a). At the instant of starting, the rotor in all dc motors is not revolving, and thus the voltage generated is zero. As a result, the terminal voltage is applied across the relatively small armature resistance. Consequently, a very large current flows through the conductors of the armature. Mathematically, the armature's starting current ($I_{a_{st}}$) is

$$I_{a_{st}} = \frac{V_t}{R_a} \tag{6.68}$$

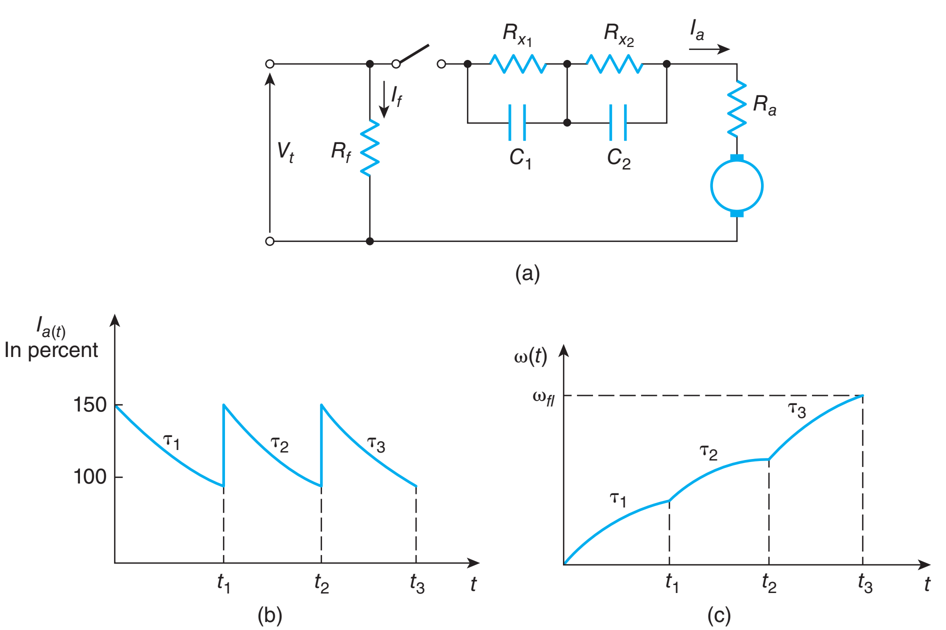

FIG. 6-21 Starting a dc shunt motor through step-by-step reduction of the external armature resistors: **(a)** equivalent circuit, **(b)** armature current versus time, **(c)** speed versus time.

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High starting current is undesirable because it adversely affects the dc power supply, the commutation process, and the rate of heat loss within the machine.

In order to limit excessively large starting currents, dc shunt motors—like all other types of dc machines—are provided with external resistors that are connected in series with the armature circuit (see Fig. 6-21(a)). During starting, these resistors are shorted,* in steps, by the contactors (C_1 and C_2) until the armature current reaches its rated value. At the same time, the speed of the motor increases from zero up to its nominal value.

The variations in armature current and speed, as functions of time during the step-by-step reduction in the external resistors for a particular motor, are shown in Figs. 6-21(b) and (c), respectively.

At starting, both of the contactors (see Fig. 6-21(a)) are open. Therefore, the armature current is limited to

$$I_{a_{st}} = \frac{V_t}{R_a + R_{x_1} + R_{x_2}} \tag{6.69}$$

As soon as the motor starts to rotate, the armature voltage increases. Consequently, the armature current is gradually reduced. From KVL, we have

$$I_a = \frac{V_t - K_1\omega}{R} \tag{6.70}$$

where K_1 is the constant of the machine, R is the total resistance of the armature circuit, and ω is the value of the motor's speed at the instant under consideration.

When the armature current reaches its rated value, the resistance R_{x_1} is shorted through the contactor C_1. At this instant, the speed of the motor cannot change instantaneously. Thus, the armature current must instantaneously increase in order to satisfy KVL. In Fig. 6-21(a), the resistors have been selected in such a way as to limit the armature current to 150% of its rated value.

For this variation of the armature current immediately before and after the removal of the first resistor (R_{x_1}), we have

$$I_{a_r} = \frac{V_t - K_1\omega_1}{R_a + R_{x_1} + R_{x_2}} \tag{6.71}$$

and

$$1.5 I_{a_r} = \frac{V_t - K_1\omega_1}{R_a + R_{x_2}} \tag{6.72}$$

where I_{a_r} is the rated current of the armature windings and ω_1 is the speed of the motor at the instant when the resistor R_{x_1} is shorted. When the second external

*The operation of the contactors is explained in Chapter 7.

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resistor is shorted, the above equations can be rewritten with a new value for the speed of the motor. The resulting equations will yield the required values of the external resistors, and the speed of the motor, at the instant a resistor is removed.

At any instant after starting, the motor's armature current and speed, as functions of time, can be found by solving the governing differential equations. The solution of these equations gives the transient response of the dc machines, which is discussed in the Web section, Chapter 6W.

The time constants that control the decay of the armature current and the build-up of the speed (see Fig. 6-21(b)) are determined by the parameters of the motor plus load. Each time a resistor is removed, the corresponding time constant will, of course, increase.

6.1.9 Open-Circuit Characteristics and DC Generators

This section discusses the open-circuit characteristics of dc machines, their effective field current, and the various types of dc generators.

Open-Circuit Characteristic

The magnetization, or open-circuit characteristic (OCC), of a machine is obtained by driving the machine as a generator at constant speed. The terminal voltage, which is equal to the voltage generated (V_g), is recorded for various values of field current (I_f).

Figure 6-22(a) shows a laboratory set-up that may be used to obtain data to plot the open-circuit characteristic of a separately excited dc machine. The mechanical power input to the generator is equal to the sum of its mechanical losses and iron losses. The OCC of the machine is shown in Fig. 6-22(b).

At low flux densities, the magnetization characteristic (MC) is linear, owing to the constant permeability of the air gap. At higher flux densities, the MC becomes nonlinear, owing to the variations in the permeability of the magnetic material.

Alternatively, at relatively low flux densities, the reluctance of the air gap predominates, and all input mmf is used to overcome the opposition to the flux that is presented by the air gap. As the flux density or the level of magnetization increases, the reluctance of the magnetic material becomes the controlling parameter of the magnetic circuit.

The machine's equivalent magnetic circuit per pole is shown in Fig. 6-22(c). The armature reaction (A.R.)′ is expressed in equivalent field ampere-turns.

The basic relationships of Chapter 1 show that the generated voltage (V_g) is proportional to the magnetic flux density (B), and also that the magnetic field intensity (H) is proportional to the field current (I_f). That is, the ordinate and the abscissa of the MC correspond to the magnetic field's flux density and flux intensity, respectively.

The flux of a coil is proportional to the coil's current and to the permeability of the magnetic material. That is,

$$\phi = K\mu I \tag{6.73}$$

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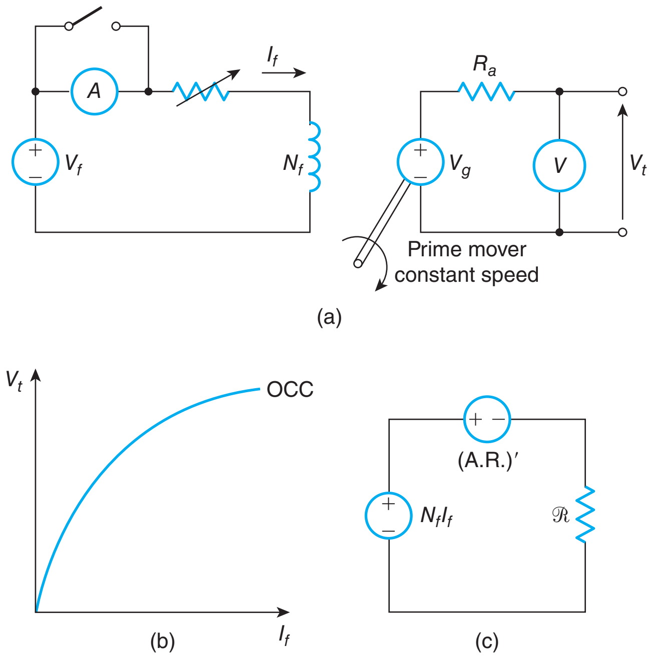

FIG. 6-22 Separately excited dc machine: **(a)** schematic for the laboratory setup used to obtain data for the magnetization curve, **(b)** typical open-circuit characteristic, **(c)** equivalent magnetic circuit per pole. (At no-load, the mmf of the armature reaction is zero.)

where K is a constant that is dependent on the physical dimensions of the magnetic material and on the winding's number of turns. As shown in Fig. 1-61, the permeability depends on the operating point or the level of magnetization.

From the generator principle, for two different speeds of operation ω_1 and ω_2, we have

$$\frac{V_{g_1}}{V_{g_2}} = \frac{\phi_1\omega_1}{\phi_2\omega_2} \tag{6.74}$$

When the effective flux ϕ_1 and ϕ_2 are the same, we obtain

$$\frac{V_{g_1}}{V_{g_2}} = \frac{\omega_1}{\omega_2} \tag{6.75}$$

In Eq. (6.75), the generated voltages V_{g_1} and V_{g_2} must be calculated at the same effective field conditions. The magnetization curve can be used in conjunction with Eq. (6.75) to find the speed of the machine for a particular operating condition.

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Usually, the generated voltage V_{g_1} is obtained from the OCC at the known speed ω_1 but at the same field condition as the generated voltage V_{g_2}. The generated voltage V_{g_2} corresponds to speed ω_2, which is normally obtained by applying KVL in the equivalent circuit of the machine under consideration. Clearly, then, the no-load characteristic can be used at full-load conditions, provided that the abscissa of the MC becomes the effective field current and its ordinate becomes the voltage generated at full-load conditions.

Effective Field Current

The general magnetic equivalent circuit per pole of a dc compound machine is shown in Fig. 6-23(a). NI is the mmf of the auxiliary windings. Referring to Fig. 6-23(b), the effective flux within the machine is given by

$$\phi = \frac{N_f I_{\text{ef}}}{\mathcal{R}} \tag{6.76}$$

where $\mathcal{R}$ is the reluctance of the path through which the flux completes its loop, and $N_f I_{\text{ef}}$ is the effective potential that magnetizes the machine.

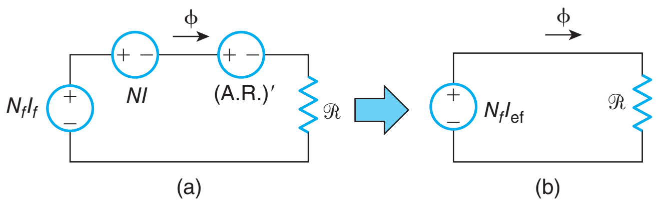

FIG. 6-23 Compound dc machine: **(a)** equivalent magnetic circuit per pole, **(b)** simplified equivalent circuit.

The effective mmf is equal to the mmf contribution of the main and auxiliary windings minus the opposition of the armature reaction. Mathematically,

$$N_f I_{ef} = N_f I_{f_a} \pm N_s I_s - (\text{A.R.})' \tag{6.77}$$

Dividing both sides by the turns of the main field winding N_f, we obtain

$$I_{ef} = I_{f_a} \pm \frac{N_s}{N_f} I_s - \text{A.R.} \tag{6.78}$$

where

$I_{f_{\text{ef}}}$ = the effective field current of the machine. Although this current cannot be seen in the machine's electrical equivalent circuit, it constitutes the abscissa of its OCC under load conditions.

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I_{f_a} = the actual current through the main field winding of the machine. This current can easily be calculated from the given data and from use of the equivalent circuit of the machine under consideration.

$N_s I_s$ = the magnetomotive force of the auxiliary field winding. This winding is a feature of a compounded machine, and its flux may aid (cumulative connection) or oppose (differential connection) the flux of the main winding.

(A.R.)′ = the armature reaction in equivalent field ampere-turns.

A.R. = the armature reaction in equivalent field amperes.

The cross-magnetizing mmf of the armature reaction is directly proportional to the armature current, but the resulting reduction in the effective, *voltage-generating flux* is approximately proportional to the *square of the armature current.*

The effects of the armature reaction on the OCC and on the full-load characteristics of a separately excited generator are shown in Fig. 6-24. In Fig. 6-24(b), the A.R. is expressed as a percentage of the flux at no-load. The letters A and B are used to identify the no-load and full-load conditions, respectively. In Fig. 6-24(c), the effects of the armature reaction are expressed in volts. This is seldom used in the analysis of problems because full-load terminal characteristics are unavailable. In Fig. 6-24(d),

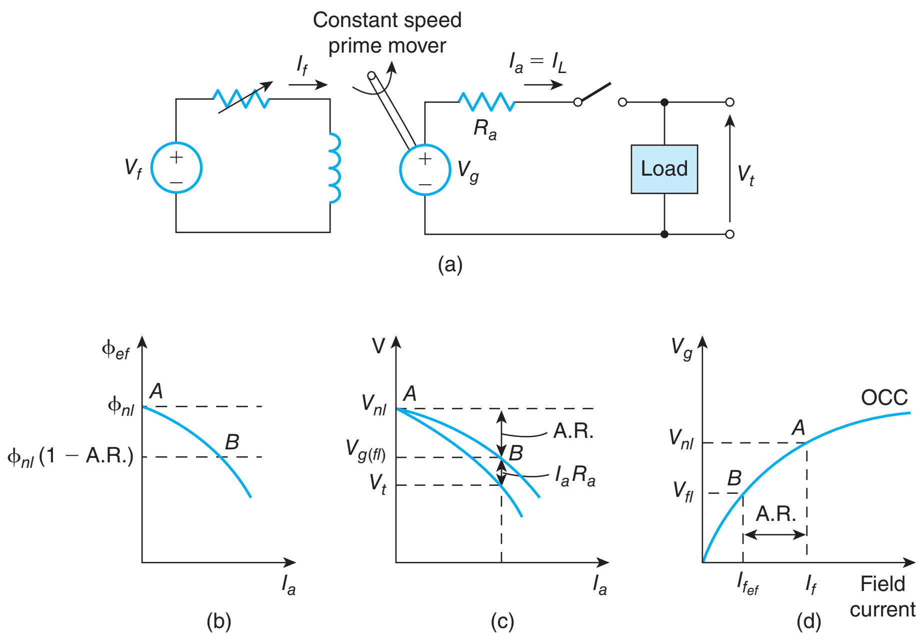

FIG. 6-24 Separately excited generator—an illustration of the alternative units of armature reaction: **(a)** equivalent circuit, **(b)** effective flux versus load current (armature-reaction effects in percent of the flux at no-load, **(c)** generated and terminal voltages as functions of load current (armature-reaction effects in volts), **(d)** open-circuit characteristic (armature-reaction effects in terms of equivalent field amperes).

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the armature reaction is expressed in equivalent field amperes, which is the most popular unit of measure of armature reaction. The open-circuit characteristic is used at no-load and at full-load conditions.

The effects, then, of the armature reaction can be expressed in terms of no-load flux, generated voltage, and/or field current.

Example 6-5

A dc series motor is operated from a 450 V supply. The total armature and series field resistance is 0.40 ohm. Its magnetization characteristic, taken at 800 r/min, is

Volts	360	412	460	470
Amperes	35	45	55	65

The demagnetizing effect of the armature reaction, which may be assumed to be proportional to the square of the armature current, is such that for an armature current of 55 A, the flux per pole is reduced by 10%. When the motor draws 65 A from the 450 V supply, determine:

a. The speed.
b. The electromagnetic torque developed.
c. The output power if the rotational losses consume 1500 W.

SOLUTION

a. The equivalent circuit of the motor is shown in Fig. 6-25(a). At 800 r/min and at 65 A from the given no-load characteristic, we obtain

$$V_{g_1} = 470 \text{ V}$$

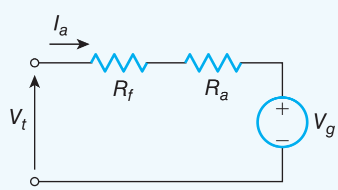

FIG 6-25(a)

This voltage corresponds to a flux of ϕ_0 webers. Owing to the armature reaction, the effective flux ϕ_{ef} is

$$\phi_{ef} = \phi_0 - \left(\frac{10}{100}\right)\left(\frac{65}{55}\right)^2 \phi_0$$
$$= 0.86\phi_0$$

From the above, the actual voltage generated at full-load conditions, but at 800 r/min, is

$$V_{g_1} = 470(0.86) = 404.36 \text{ V}$$

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The generated armature voltage at the unknown speed, but at the same flux level as V_{g_1}, is

$$\begin{aligned} V_{g_1} &= V_t - I_a R \\ &= 450 - 65(0.4) \\ &= 424 \text{ V} \end{aligned}$$

Substituting the above data into Eq. (6.75), we obtain

$$n_2 = 424\left(\frac{800}{404.36}\right) = \underline{\underline{838.87 \text{ r/min}}}$$

b. The torque developed is

$$T = \frac{V_{g_2} I_a}{\omega_2} = \frac{424(65)}{838.87\left(\frac{2\pi}{60}\right)} = \underline{\underline{313.73 \text{ N} \cdot \text{m}}}$$

c. From the power-balance equation, we have

$$P_d = P_{r_1} + P_0$$

from which

$$P_0 = 424 \times 65 - 1500 = \underline{\underline{26.06 \text{ kW}}}$$

An Alternative—But Incorrect—Solution of Part (a)

The armature voltage at the unknown speed is, as before,

$$V_{g_2} = 424 \text{ V}$$

The effective value of the field current is

$$I_{\text{ef}} = (\text{actual field current}) - (\text{armature reaction})$$

At 65 A,

$$\begin{aligned} I_{\text{ef}} &= 65 - 65\left(\frac{10}{100}\right)\left(\frac{65}{55}\right)^2 \\ &= 59.92 \text{ A} \end{aligned}$$

Projecting this current on the OCC (Fig. 6-25(b)), we obtain

$$V_{g_1} = 466 \text{ V}$$

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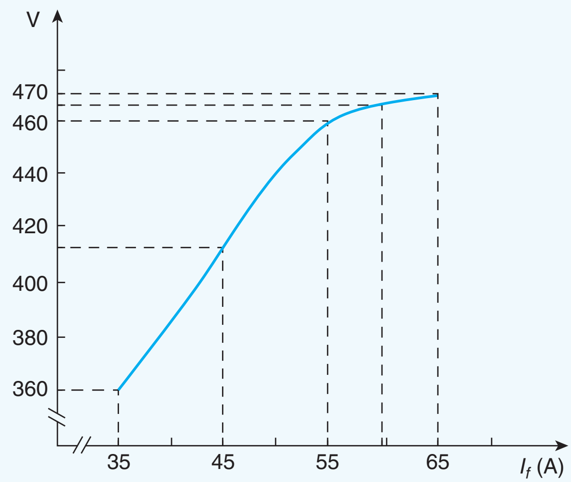

FIG 6-25(b)

and

$$n_2 = 424\left(\frac{800}{466}\right)$$
$$= 727.90 \text{ r/min}$$

which is lower than the speed already calculated by an amount that is equal to

$$\frac{838.87 - 727.9}{727.9}(100) = 15.25\%$$

In other words, it is a mistake to assume that the effective flux is dependent only on the field current. The change of permeability, and its effects on the voltage generated, cannot be assumed to be negligible.

Exercise 6-6

Given the following data for a dc motor that operates at saturation:

Condition	*Generated Voltage (V)*	*Effective Field (A)*	*Speed (r/min)*
1	213	1.4	900
2	208	1.2	?

a. Explain why the tabulated data, when substituted into Eq. (6.75), will not give the speed of the motor at operating condition 2.

b. What additional information is required to calculate the unknown speed?

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Shunt Generators

Figures 6-26(a) and (b) show, respectively, the equivalent circuit and the OCC of a shunt-type generator. The shunt generator does not require any excitation for starting or operating. It is self-excited. This advantage results from the machine's residual magnetism—a characteristic of the magnetic material—which provides the required field. As soon as the prime mover starts to turn the rotor, a voltage will be generated in the armature windings because they are rotated through the field of the stator's residual magnetism.

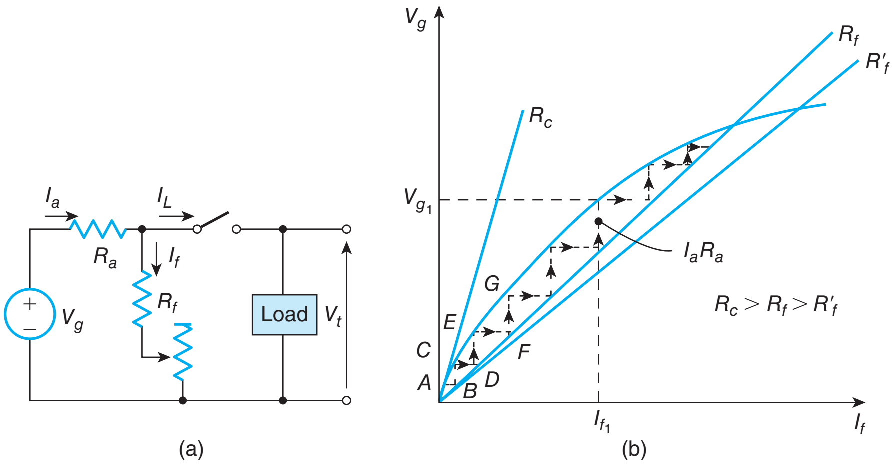

FIG. 6-26 Shunt generator: **(a)** equivalent circuit, **(b)** magnetization characteristic illustrating the build-up voltage for the no-load conditions.

Refer to Fig. 6-26(b). When the armature starts to rotate, a small voltage will be generated, as indicated by point A. This voltage, impressed across the field circuit (point B), will cause a current to flow. If the polarity of the connection is correct, the mmf of this current will reinforce that of the residual magnetism; the strengthened magnetic field will then generate the voltage corresponding to point C, which in turn will increase the field current to the value corresponding to point D. The build-up process (*positive feedback*) will continue until the saturation curve intersects the field-resistance line. The slope of this line is equal to the total resistance of the field winding (resistance of the winding plus the portion of the external adjustable resistor that may be incorporated). The intersection of the magnetization curve and the field resistance line furnishes the operating point at no-load.

For a particular field current I_{f_1}, the generated voltage and the terminal voltage are given respectively by

$$V_{g_1} = I_{f_1}R_f + I_{a_1}R_a \tag{6.79}$$

$$V_{t_1} = I_{f_1}R_f \tag{6.80}$$

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From the above it is evident that the vertical lines drawn from the OCC to the field-resistance line represent the voltage drop across the armature resistance. The longest of these lines will give—indirectly—the maximum permissible value of armature current at rated speed.

By decreasing the field resistance (R_f') below its rated value, the no-load voltage will increase slightly. Conversely, by increasing the field resistance, the no-load voltage will decrease. The value of the field resistance, at which the build-up of the generated voltage cannot take place, is designated as the critical field resistance (R_c).

In general, the field resistance controls the starting, the terminal voltage, and the voltage regulation of the shunt generator.

Compound dc Generators

The equivalent circuits and the terminology of compound dc generators are similar to those of dc compound motors described in Section 6.1.7. The only differences between a compound dc generator and a compound dc motor are the energy flow and the eternal characteristics of the machine. The external characteristic of a motor is its torque-speed curve, while for a generator, it is its terminal voltage versus its load-current curve.

Figure 6-27(a) shows an elementary representation of a machine's magnetic pole on which the main and auxiliary windings are identified. The auxiliary winding is often referred to as the series winding to differentiate it from the shunt field winding.

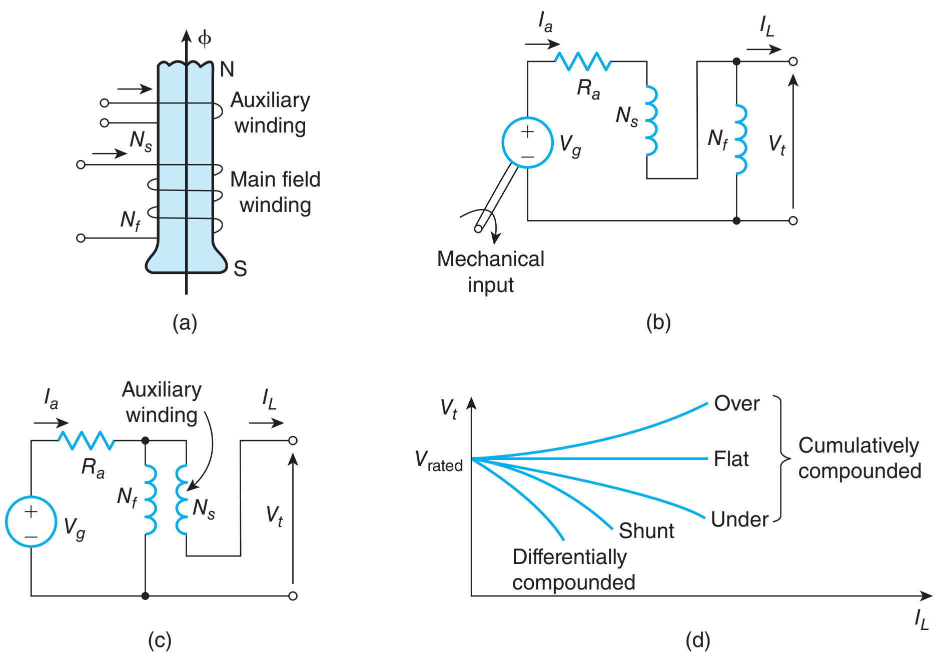

FIG. 6-27 Compounded dc generators: **(a)** physical location of the main and auxiliary field windings, **(b)** long-shunt, **(c)** short-shunt, **(d)** characteristics of differentially and cumulatively compounded generators.

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The equivalent circuits of a long-shunt and a short-shunt compounded generator are shown in Figs. 6-27(b) and (c), respectively. The external characteristics of the compounded generators are depicted in Fig. 6-27(d). The cumulatively compounded dc generators are further classified as overcompounded, flat-compounded, or undercompounded, depending on whether the output voltage increases, remains constant, or decreases as the load current increases.

Figure 6-28(a) shows the equivalent circuit of a short-shunt differentially compounded dc generator. Figure 6-28(b) shows its OCC and the effects of both the auxiliary winding and the armature reaction on the magnetization of the machine.

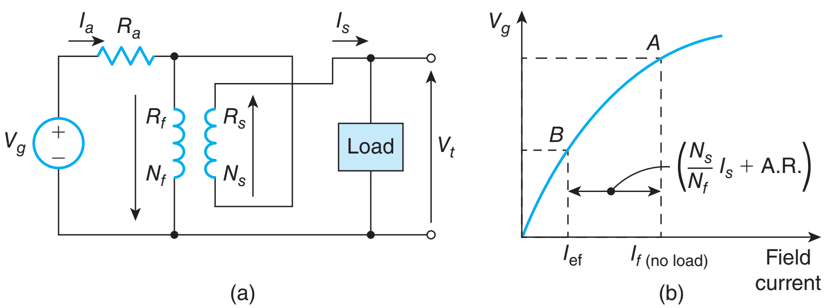

FIG. 6-28 Short-shunt differentially compounded generator: **(a)** equivalent circuit, **(b)** open-circuit characteristic.

EXAMPLE 6-6

A long-shunt, cumulatively compounded dc generator has 1800 shunt-field turns per pole and 12 series-field turns per pole. The armature resistance is 0.12 ohm, the series-field resistance is 0.04 ohm, and the shunt-field resistance is 210 ohms. Data for the open-circuit characteristic at 1700 r/min is as follows:

Volts	110	200	280	300
Amperes	0.5	1.1	1.9	2.35

The generator supplies a load of 16 kW at 215 V when it is driven at 1700 r/min. Determine the speed at which the generator must be driven to supply a load of 23 kW at 270 V. Assume armature reaction is proportional to the armature current squared.

SOLUTION

The equivalent circuit of the long-shunt, cumulatively compounded dc generator is shown in Fig. 6-29(a). The series-field winding, as the name of the generator

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implies, aids the shunt-field winding. It is for this reason that the field currents flow in the same direction in the equivalent circuit. The OC characteristic is shown in Fig. 6-29(b). This characteristic can be used to obtain the no-load and full-load machine voltages and field currents only at 1700 r/min.

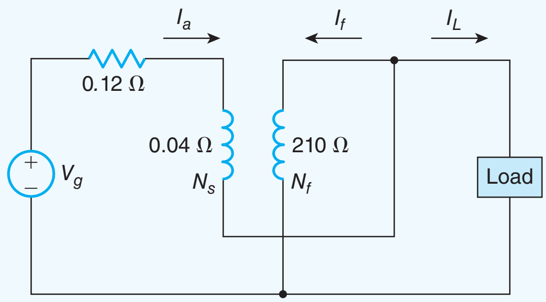

FIG. 6-29(a)

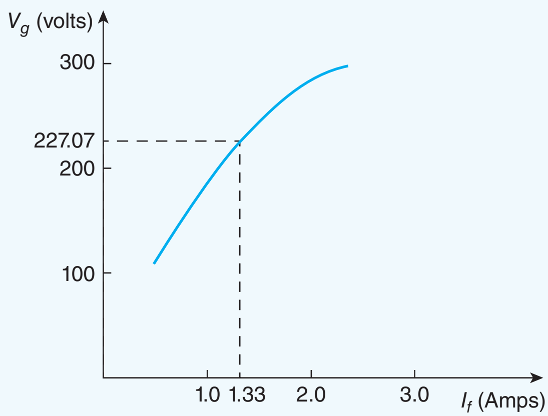

FIG. 6-29(b)

Load Condition: 16 kW at 215 V

The given data will be used to calculate the armature reaction as follows. Referring to Fig. 6-29(a), we have

$$\text{load current} = I_L = \frac{P}{V} = \frac{16{,}000}{215} = 74.42 \text{ A}$$

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The current through the main field winding is

$$I_f = \frac{215}{210} = 1.02 \text{ A}$$

The armature current is

$$\begin{aligned} I_a &= I_L = I_f \\ &= 74.42 + 1.02 \\ &= 75.44 \text{ A} \end{aligned}$$

The generated voltage is

$$\begin{aligned} V_g &= V_t + I_a(R_a + R_s) = 215 + 75.44\,(0.12 + 0.04) \\ &= 227.07 \text{ V} \end{aligned}$$

Projecting this voltage on the given OCC, we obtain the effective field current:

$$I_{\text{ef}} = 1.33 \text{ A}$$

The armature reaction—at an armature current of 75.44 A—is found by using Eq. (6.78). Substituting the known data, we obtain

$$1.33 = 1.02 + \frac{12}{1800}(75.44) - \text{A.R.}$$

from which

$$\text{A.R.} = 0.197 \text{ equivalent field amperes}$$

Load Condition: 23 kW at 270 V

The load current at the unknown speed is

$$I_L = \frac{23{,}000}{270} = 85.18 \text{ A}$$

The current through the shunt field is

$$I_{f_a} = \frac{270}{210} = 1.28 \text{ A}$$

The armature current is

$$I_a = 85.18 + 1.28 = 86.47 \text{ A}$$

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From the statement of the problem, the new armature reaction is

$$\text{A.R.} = 0.197\left(\frac{86.47}{75.44}\right)^2$$
$$= 0.258 \text{ equivalent field amperes}$$

Using Eq. (6.78), we find that the new effective field current is

$$I_{\text{ef}} = 1.28 + \frac{12}{1800}(86.47) - 0.258$$
$$= 1.60 \text{ A}$$

Projecting this current on the given OCC gives the armature voltage at 1700 r/min. From the curve,

$$V_{g_1} = 255 \text{ V}$$

At the same field condition, but at the unknown speed, the generated voltage is obtained from the equivalent circuit as follows:

$$V_{g_2} = 270 + 86.47(0.12 + 0.04) = 283.84 \text{ V}$$

The required speed is found by using Eq. (6.75). Substituting, we obtain

$$\frac{283.84}{255} = \frac{n_2}{1700}$$

from which

$$n_2 = \underline{\underline{1892.24 \text{ r/min}}}$$

In other words, the prime mover must rotate at 1892.24 r/min to meet the new load requirements.

For comparison purposes, the results are summarized in Table 6-3.

TABLE 6-3 Summary of results of Example 6-6

Condition	Output: Power (kW)	Output: Voltage (V)	Speed (r/min)	Shunt Field (A)	Effective Field (A)	Armature Reaction (A)
1	16	215	1700	1.02	1.33	0.197
2	23	270	1892.24	1.28	1.6	0.258

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6.2 Modern Methods of Speed Control

6.2.1 Rectifiers

The most efficient and accurate method of controlling the starting current and the torque-speed characteristics of a dc motor is by using a variable armature and field voltage. The variable dc voltage is obtained through controlled rectifiers. Controlled rectifiers are used, as shown in Fig. 6-30(a), to convert single or polyphase ac voltage sources to dc-controlled voltage supplies.

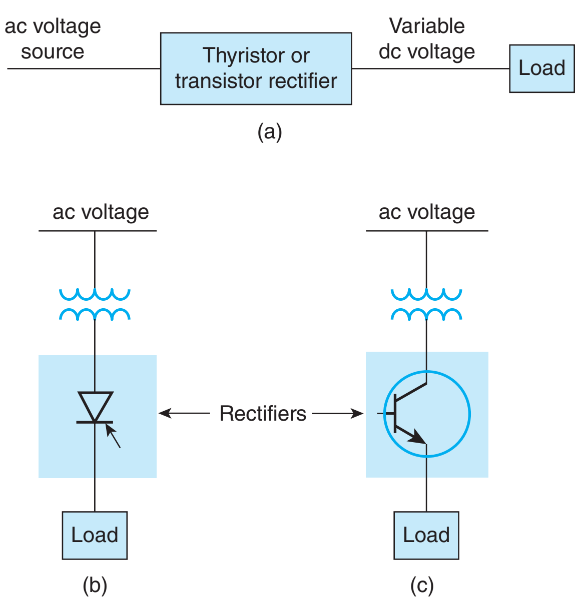

Fig. 6-30 AC-to-DC conversion: **(a)** block diagram representation, **(b)** one-line diagram representation, using thyristors, **(c)** one-line diagram representation, using transistors.

The controlling device for dc machines with low ratings is usually the transistor or the *I*GBT (Fig. 6-30(c)), while for higher machine ratings, it is the thyristor* (Fig. 6-30(b)). Chapter 3 discusses use of the transistor as a control switch. In this section, only the thyristor-rectifying circuits will be analyzed. The dc voltage that these circuits produce at their output terminals can be controlled smoothly by properly varying the thyristor's firing angle.

Controlled rectifiers, however, distort the power system's voltage waveforms, generate harmonics, and always result in an overall power factor of

*The basic operation of transistors and thyristors is discussed in the Appendix of the online Section DW.4.

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negative nature. The harmonics may be reduced or eliminated completely by using suitable filters, and the power factor can be improved by adding equipment that generates lagging (leading-power-factor) kVAR. Of course, this significantly increases the overall cost of the rectifying unit and decreases its reliability.

In general, a controlled rectifier system consists of an isolating or a voltage step-down power transformer, a thyristor bridge network, and various other control and protection apparatuses.

In the next section the full-wave, single-phase rectifiers and the three-phase controlled rectifiers are discussed briefly. These rectifiers are also known, respectively, as two-pulse and six-pulse rectifiers because of the number of output voltage pulses they produce during one complete cycle of the input voltage.

6.2.2 Single-Phase, Full-Wave Controlled Rectifiers

A single-phase, full-wave controlled rectifier is shown in Fig. 6-31(a). The resulting pertinent waveform for a firing angle of less than 90 degrees is shown in Fig. 6-31(b). The gating of the thyristors is synchronized to the supply voltage, and Q_1 and Q_2 are pulsed at a firing angle of α radians. The gating of Q_3 and Q_4 is initiated at a firing angle $\alpha + \pi$ radians.

The load current is continuous: That is, current flows continuously over a complete cycle. As a result, one pair of thyristors is always conducting. Continuity results because thyristors Q_3 and Q_4 are turned ON before the current through Q_1 and Q_2 is reduced to zero. Continuity of current depends on the applied voltage, the thyristor's firing angle, and the impedance of the load. In an inductive impedance, the current reaches its zero value sometime after the applied voltage reaches its zero value. Thus, although the voltage increases negatively, the thyristor is not switched OFF until its current becomes zero.

With regard to Fig. 6-31(b), the average value of the output voltage is obtained as follows:

$$V_{\text{av}_o} = 2\left(\frac{1}{2\pi}\right)\int_{\alpha}^{\pi+\alpha} V_m \sin \omega t \, d\omega t \qquad \textbf{(6.82)}$$

from which

$$V_{\text{av}_o} = 2\,\frac{V_m}{\pi}\cos\alpha \qquad \textbf{(6.83)}$$

As can be seen from Eq. (6.83), the average value of the dc voltage can be varied smoothly by changing the firing angle of the thyristors accordingly. The average value of the output voltage, as a function of the angle of retard, is sketched in Fig. 6-32. At $\alpha = \pi/2$ radians, the dc value of the output voltage is equal to zero, and a further increase in the firing angle will result in negative output dc voltage.

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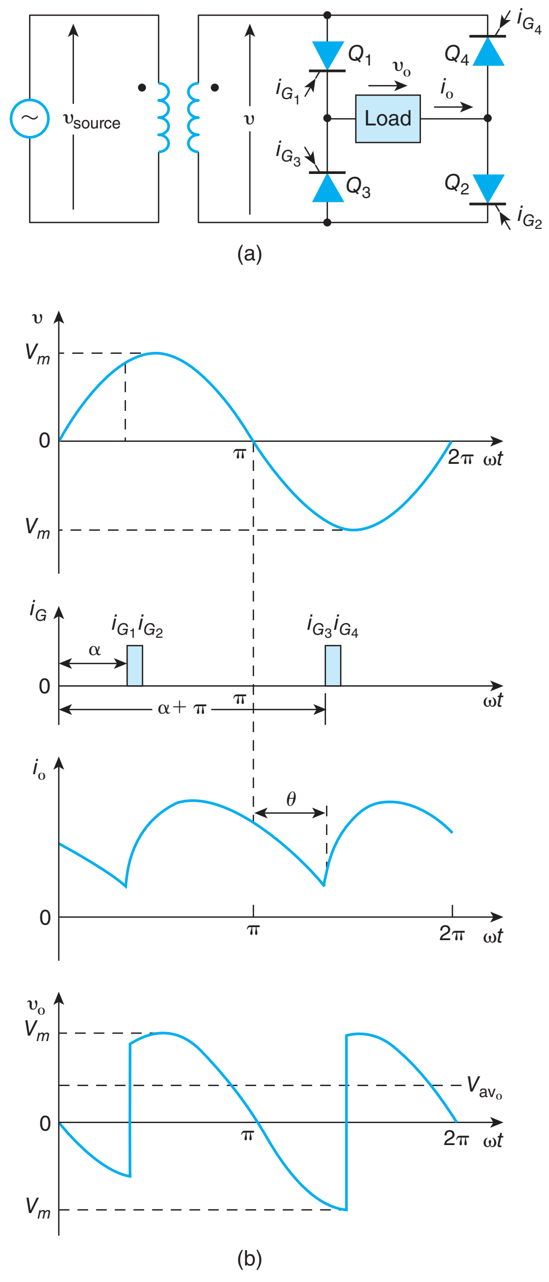

FIG. 6-31 Single-phase, full-wave controlled rectifier continuous load current: **(a)** circuit, **(b)** pertinent waveforms.

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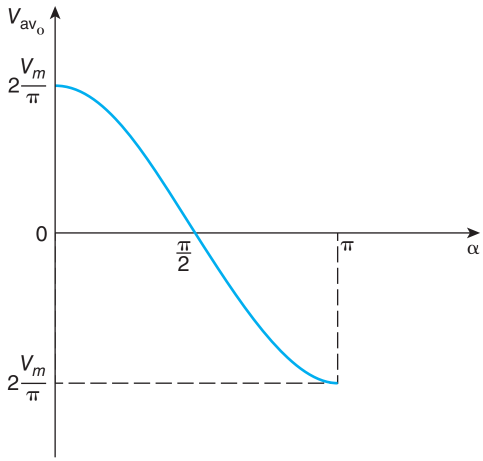

FIG. 6-32 Average value of output voltage versus angle of retard for a single-phase, full-wave controlled rectifier.

Example 6-7

A separately excited dc motor is supplied from a 208 V source through a thyristorized, single-phase, full-wave rectifier. The motor's armature resistance is 0.10 ohm. When the firing angle of the thyristors is 40 degrees, the motor draws 100 A and operates at 950 rpm. When the field current is reduced by 20%, what should be the firing angle so that the speed and armature current remain constant?

Solution

At the given condition and for a constant field current, we have

$$V_t = I_a R_a + E_a$$

$$E_a = K_1 \phi \omega = K\omega$$

$$\frac{2\sqrt{2}}{\pi}(208)\cos 40 = 100\,(0.1) + K950\left(\frac{2\pi}{60}\right)$$

from which

$$K = 1.3415 \text{ V/rad/sec}$$

When the field is reduced by 20%, we have

$$\frac{2\sqrt{2}}{\pi}(208)\cos\alpha = 100(0.1) + 0.8(1.3415)\left(950\,\frac{2\pi}{60}\right)$$

from which

$$\alpha = \underline{\underline{51.4 \text{ degrees}}}$$

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Exercise 6-7

A separately excited dc motor is supplied from a 208 V source through a thyristorized, single-phase, full-wave rectifier. The motor's armature resistance is 0.10 ohm. When the firing angle of the thyristors is 40 degrees, the motor draws 100 A and operates at 950 rpm. When the firing angle is reduced by 20%, by how much should the field current be increased so that the speed of the motor and its armature current will remain constant?

Answer 1.115

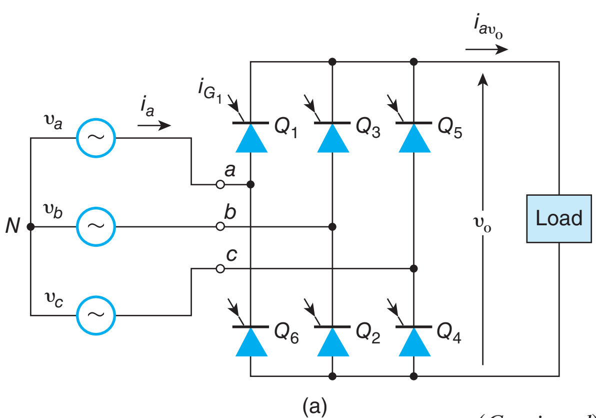
6.2.3 Three-Phase, Full-Wave Controlled Rectifiers

A three-phase, full-wave controlled rectifier is shown in Fig. 6-33(a). This circuit is commonly known as a 3-ϕ bridge rectifier, a Graetz rectifier, or simply a six-pulse rectifier. Each of the output voltage pulses has a fundamental frequency, which is six times that of the supply voltage. The impedance Z_n of the R-L load to the nth harmonic current is

$$Z_n = R + jn\omega L \tag{6.84}$$

where $n = 6$, 12, and so on, and ω is the angular rotation of the source voltage. Owing to increased impedances, the various harmonic components of the load current are much smaller than the corresponding ones produced by a two-pulse controlled rectifier.

Analysis of a six-pulse rectifier is simplified by considering the effects of the positive and negative line voltages, as shown in Fig. 6-33(b). During one complete cycle, the controlling range of each of these voltages is given by the interval through which the particular voltage has, relatively, the maximum positive value. For example, as can be seen in Fig. 6-33(b), the controlling range of the voltage v_{ab}



(*Continued*)

FIG. 6-33 Three-phase, full-wave rectifier: **(a)** circuit.

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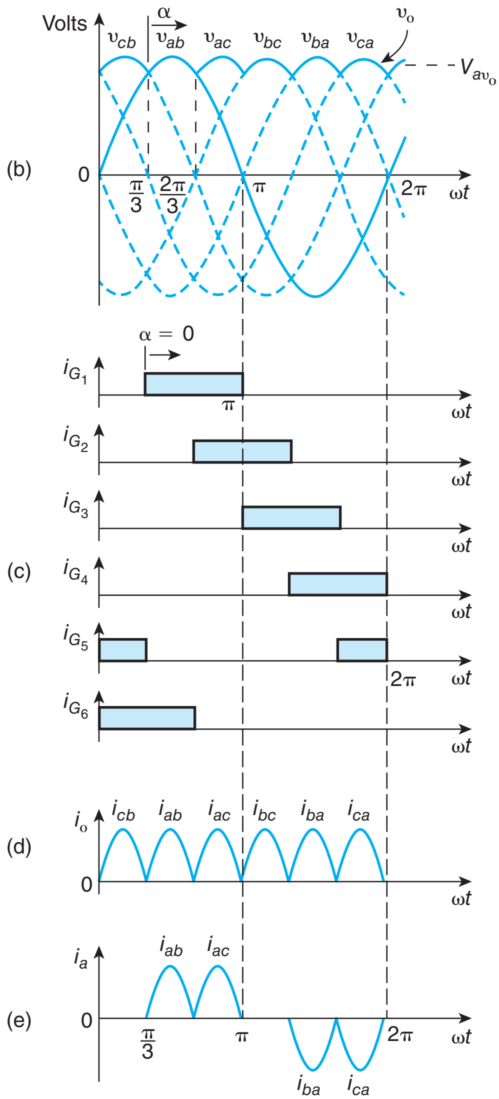

FIG. 6-33 (*Continued*) **(b)** supply voltages, **(c)** thyristors' firing pulses, **(d)** output current waveform for resistive load and $\alpha = 0$, **(e)** current pulses through supply line "a."

starts at $\pi/3$ radians and goes up to $2\pi/3$ radians. The angle of retard for thyristor Q_1 is thus measured starting at $\pi/3$ radians. Therefore, for thyristor Q_1, Eq. (6.85) where, as shown in Fig. 6-33(b), ωt is the angle on which the supply voltages depend.

$$\alpha = \omega t - \frac{\pi}{3} \qquad (6.85)$$

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The "turning ON" of the other thyristors is synchronized to the supply frequency, and their firing angles are spaced $\pi/3$ radians apart. The firing pulse of each thyristor is shown in Fig. 6-33(c). The end of conduction, as in all other circuits using thyristors, depends on the applied voltage and the load impedance. For rectifier operation, the angle of retard must be restricted to less than $\pi/2$ radians.

Figure 6-33(d) shows the output current pulses for a firing angle of zero degrees. A firing angle for a particular thyristor indicates the time when the thyristor is gated relative to its corresponding line-to-line voltage.

Figure 6-33(e) shows the output current pulses conducted through supply line "a." Then in the six-pulse circuit, each supply line carries four of the six output current pulses in an alternating symmetry and thus has no dc component. This permits the use of a 3-ϕ transformer to supply the voltage required by the bridge. Furthermore, the line-current waveform, as can be shown by Fourier series analysis, contains a fundamental component and various harmonics. The harmonics transport no real power to the load and distort the upstream current waveforms. This may lead to original text resonance problems and communication interference.

The type and duration of the output current pulses depend on the firing angle and the parameters of the load. The limits of integration for calculating the various parameters depend on whether the output current is continuous or discontinuous.

For continuous load current, the dc value (V_{av_o}) of the output voltage is

$$V_{\text{av}_o} = \frac{6}{2\pi}\int_{(\pi/3)+\alpha}^{(2\pi/3)+\alpha} V_m \sin \omega t\,(d\omega t) = 3\,\frac{V_m}{\pi}\cos\alpha \tag{6.86}$$

When the load to a six-pulse rectifier is a dc machine, Eq. (6.86) gives the motor's terminal voltage. Neglecting the resistance of the armature winding, from the generator principle and from KVL, we have

$$V_g \approx 3\,\frac{V_m}{\pi}\cos\alpha = K\phi\omega \tag{6.87}$$

Thus, the speed of the motor, as a function of the firing angle, is

$$\omega \approx \frac{3V_m}{K\phi\pi}\cos\alpha \tag{6.88}$$

That is, for constant field current, the speed of the motor is directly proportional to the cosine of the firing angle. Therefore, it is evident that the thyristor's firing angle controls the motor's starting current and speed of operation.

Refer to Fig. 6-33(e). By assuming the load current is constant at I_{av_o}, we find the rms value of the supply line currents as follows:

$$I_{\text{rms}}^2 = \frac{2}{2\pi}\int_{\pi/3}^{\pi} I_{\text{av}_o}^2\, d\omega t \tag{6.89}$$

$$= \frac{2}{3}\, I_{\text{av}_o}^2 \tag{6.90}$$

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From Eq. (6.90),

$$I_{\text{rms}} = \sqrt{\frac{2}{3}}\, I_{\text{av}_o} \tag{6.91}$$

Equation (6.91) gives the rms value of the line current, not of its fundamental component.

The apparent power of the supply source is

$$|S| = 3V_{an}I_a = 3\,\frac{V_{L\text{-}L}}{\sqrt{3}}\sqrt{\frac{2}{3}}\, I_{\text{av}_o} = \sqrt{2}\, V_{L\text{-}L}\, I_{\text{av}_o} \tag{6.92}$$

where $V_{L\text{-}L}$ is the rms value of the line-to-line supply voltage. Equation (6.92) can be used to size the isolating transformer, which supplies power to the rectifying bridge.

The apparent or total power factor ($\cos\theta_T$) at the input terminals of the bridge is found as follows:

$$\text{power} = \sqrt{3}\, V_{L\text{-}L}\, I_L \cos\theta_T \tag{6.93}$$

from which

$$\cos\theta_T = \frac{V_{\text{av}_o} I_{\text{av}_o}}{\sqrt{3} V_{L\text{-}L} I_L} \tag{6.94}$$

By substituting for V_{av_o} and I_L their equivalent expressions (given, respectively, by Eqs. (6.86) and (6.91)), we obtain

$$\cos\theta_T = \frac{3\,\dfrac{V_m}{\pi}\cos\alpha I_{\text{av}_o}}{\sqrt{3}\,\dfrac{V_m}{\sqrt{2}}\sqrt{\dfrac{2}{3}}\, I_{\text{av}_o}} \tag{6.95}$$

From the above,

$$\cos\theta_T = \frac{3}{\pi}\cos\alpha \tag{6.96}$$

The line current to the input terminals of the bridge has a fundamental component and various harmonics. Since the harmonics transport no average power to the load, the power measured with a nondigital power meter is

$$P = \sqrt{3} V_{L\text{-}L} I_L \cos\theta \tag{6.97}$$

where I_L is the rms value of the fundamental component of the line current and $\cos\theta$ is the apparent power factor of the circuit. From Eqs. (6.93) and (6.97),

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we obtain

$$\cos \theta_T = g \cos \theta \qquad \textbf{(6.98)}$$

where g is the ratio of the rms value of the fundamental component of the line current divided by the rms value of the current waveform.

EXAMPLE 6-8

The Δ-Y transformer shown in Fig. 6-34(a) supplies power to a six-pulse bridge rectifier. The line-to-line voltages across the secondary lines of the transformer are as shown in Fig. 6-34(b). The load current is constant and the transformer is ideal, with the turns ratio equal to unity. For a firing angle of zero degrees, draw the waveforms of the current through:

a. The load.

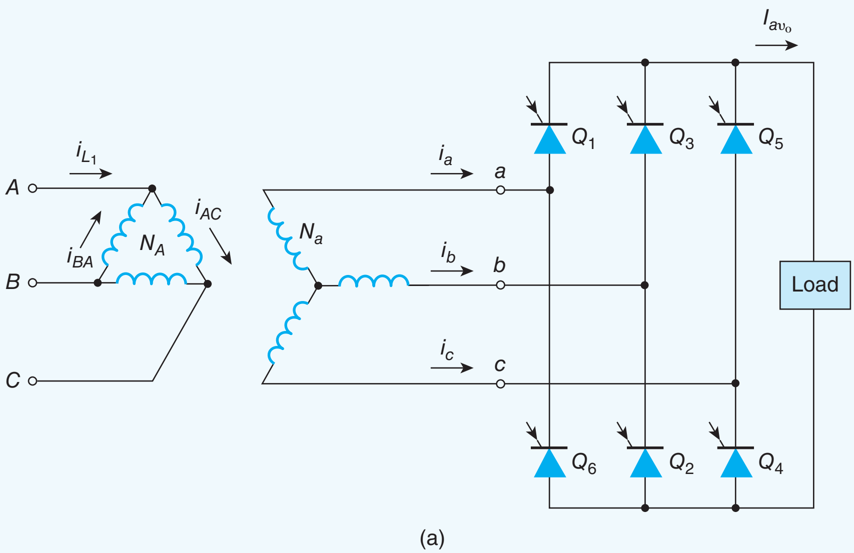
b. The transformer windings.

c. The primary supply line $\left(i_{L_1}\right)$.

SOLUTION

a. The waveform of the load current I_{av_o} is shown in Fig. 6-34(c). This current is continuous and of constant amplitude because of the load's inductance.

b. Each of the secondary transformer lines conducts during only two-thirds of the applied voltage's cycle. Thus, for continuous load current, each secondary transformer line conducts for $\frac{2}{3}\pi$ radians in the positive direction and $\frac{2}{3}\pi$



(*Continued*)

FIG. 6-34 Six-pulse rectifier. Waveforms of the currents through the load, the secondary windings, and a primary line of a Δ-Y transformer.

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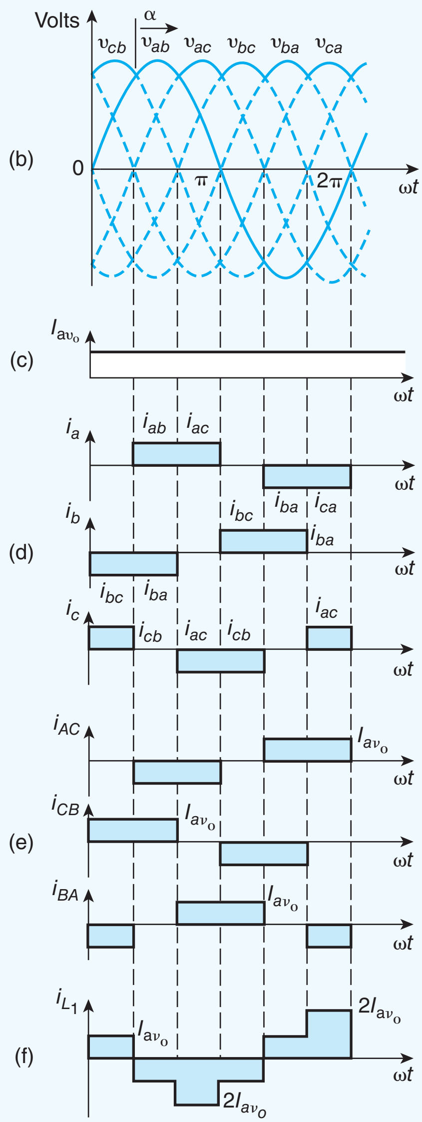

FIG. 6-34 (*Continued*)

radians in the negative direction. The particular conducting period of each secondary transformer line can be obtained by inspection of the given line-to-line voltage waveforms. For example, line "a" conducts in the positive direction when voltages v_{ab} and v_{ac} are the highest voltages in the circuit, and conducts in the negative direction when voltages v_{ba} and v_{ca} have the highest

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potential in the circuit. Therefore, line "a," as shown in Fig. 6-34(d), conducts only during the time intervals between 60 to 180 degrees and 240 to 360 degrees. The currents in the other secondary lines (i_b and i_c) are drawn as shown in Fig. 6-34(d).

The transformer's primary winding currents are obtained from the secondary line currents by using KVL in magnetic circuits. That is,

$$N_A i_{AC} = N_a\, i_a$$

Since the numbers of primary and of secondary winding turns are equal, the magnitude of the winding currents must also be equal. However, these currents must be in opposite directions, as per Lenz's law.

In a similar manner, the currents through the other primary transformer windings can easily be determined. The resulting waveforms are shown in Fig. 6-34(e).

c. The primary line current is found by applying KCL in the delta-connected winding. Considering line "A," we have

$$i_{L_1} = i_{AC} - i_{BA}$$

From the above, the current through line "A" is obtained graphically by superimposing the negative of i_{BA} on the winding current i_{AC} (See Fig. 6-34(f).)

EXAMPLE 6-9

A 20 kW, 220 V, 1200 r/min, 0.94 efficient dc motor is supplied through a six-pulse rectifier with a constant current. The line-to-line voltage across the secondary terminals of the transformer is 208 V. Determine:

a. The nominal firing angle of the thyristors.
b. The apparent power factor of the load, as seen from the secondary terminals of the rectifying transformer.
c. The line current toward the rectifying bridge.
d. The kVA rating of the rectifying transformer.

SOLUTION

a. The thyristor's firing angle is found from Eq. (6.86). Solving for α and substituting the given values, we obtain

$$\alpha = \arccos \frac{(220)\pi}{3\sqrt{2}\ 208}$$
$$= \underline{\underline{38.45^\circ}}$$

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b. From Eqs. (6.96), we have

$$\cos\theta_T = \frac{3}{\pi}\cos\alpha$$
$$= \frac{3}{\pi}\cos 38.45$$
$$= \underline{\underline{0.75 \text{ lagging}}}$$

c. The average value of the load current is

$$I_{\text{av}_o} = \frac{P}{V_{\text{av}_o}} = \frac{20}{0.94(0.220)}$$
$$= 96.71 \text{ A}$$

Thus, from Eq. (6.91), the rms value of the line current is

$$I_{\text{av}_o} = \sqrt{\frac{2}{3}}\, 96.71$$
$$= 78.96 \text{ A}$$

d. The rms value of the load current is equal to its dc value. Thus, from Eq. (6.92), the kVA rating of the transformer is

$$|S| = \sqrt{2}\,(0.208)(96.71) = \underline{\underline{28.45 \text{ kVA}}}$$

For practical purposes, a 30 kVA, commercially available transformer should be used.

Exercise 6-8

a. Draw the output voltage and current waveforms of a single-phase, full-wave rectifier. Assume that the load current is continuous and $\alpha = 30°$.
b. A six-pulse, three-phase thyristorized rectifier supplies 100 A dc to a load. Determine the average and rms values of a thyristor's current.

Answer 33.33 A, 57.74 A

Exercise 6-9

A separately excited, 100 kW, 480 V, 0.9 efficient dc motor has an armature resistance of 0.10 Ω. The motor receives its dc supply through a six-pulse rectifier as shown in Fig. 6-35. Determine:

a. The motor's current under nominal operating conditions.

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b. The thyristor's firing angle under nominal operating conditions and at starting if the motor's starting current is limited to 800 A.

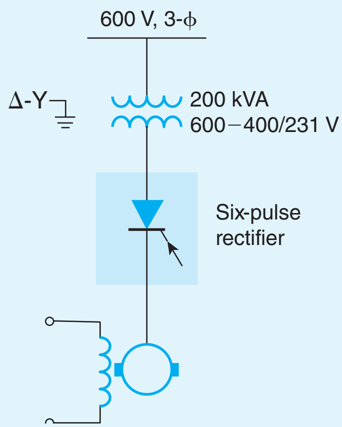

FIG. 6-35

Answer (a) 231.48 A; (b) 27.3°, 81.5°

6.3 Conclusion

The operation of a dc motor is based on the tendency of a current-carrying conductor—when placed in an external magnetic field—to rotate so that the field it creates is parallel to, and in the same direction as, the external field.

The operation of dc generators is based on the induction principle, according to which a voltage is induced in conductors that are rotated through an external magnetic field.

In their basic form, dc machines have a field coil and an armature winding. The field coil is wound around the machine's stationary magnetic poles, while the armature winding is wound on its rotating shaft. The armature current is conducted from the external motor terminals to the stationary brushes, then to the rotating segments of the commutator, and from there to the armature conductors.

The rotated armature conductors successively cut the fields of the north and south magnetic poles. As a result, the generated armature current is of alternating waveform. However, as seen from the machine's external terminals, the armature current is of dc waveform because it is rectified by the commutator-brush action. The process by which the armature current reverses direction during half the period of its cycle is called commutation.

During part of the rectification process, each armature conductor is shorted through the brushes. As a result, sparking may take place. Sparking and flashover used to be the main limitations of dc machines. However, in recent years, the

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design of the machines has been improved, and the commutation process is now sparkless. Nevertheless, the brushes require periodic replacement. In general, this process increases the maintenance cost of dc machines.

The flux of the armature current effectively reduces the magnetic field and distorts it in the interpole region. This interaction between the flux of the field and armature current is commonly known as armature reaction. Armature reaction adversely affects the characteristics of the machine and the commutation process.

All large dc machines are equipped with *compensating* and *commutating windings*. These windings are connected in series with the armature coil. Their function is to cancel the *demagnetizing* and *field-distorting* effects of the armature current. The compensating winding is placed on the pole face of the machine and cancels the reduction of the field flux. The commutating coil is wound around the narrow poles in the interpole region and cancels the distorting effect on the field-flux distribution.

Besides the main field winding, compound dc machines have an auxiliary field winding. Depending on its connection, the auxiliary field winding may aid (cumulative connection) or oppose (differential connection) the flux of the main field. The function of the auxiliary winding is to modify the external characteristics of the machine.

The characteristics of dc machines can easily be adjusted through armature or field-variable resistors or through variations of their dc voltage supply. The use of potentiometers results in higher energy losses. The use of variable dc voltage sources results in lower energy losses and in smoother torque-speed characteristics.

All modern dc drives are equipped with controlled rectifiers through which the motor characteristics are easily controlled. The six-pulse bridge rectifier is the most popular compromise between cost and desired characteristics. In general, controlled rectifiers are reliable and highly efficient, generate harmonics toward their ac voltage supply and a ripple voltage toward their dc output lines, and require kVAR compensation.

Owing to their efficiency, variable-speed dc drives have gradually displaced the more inefficient drives, such as eddy-current variable-speed drives, fluid couplings, the classical generator-motor set, and induction motors that drive fans whose flow is controlled through dampers.

Table 6-4 summarizes the essential relationships of dc machines and their control apparatus. Table 6-5 gives manufacturer's data for a 300 kW dc motor. Table 6-6 lists typical component losses of dc drives.

TABLE 6-4 Summary of important equations

Item	*Description*	*Remarks*
1	The motor principle	
	$T = K\phi I_a$	Eq. (6.13)
2	$K = \dfrac{Np}{\beta\pi}$	Eq. (6.14)

(*Continued*)

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TABLE 6-4 *(Continued)*

Item	Description	Remarks
3	The generator principle	
	$V_g = K\phi\omega$	Eq. (6.22)
4	Mutual coupling	
	$V_g = \frac{p}{2}\omega I_f L_{\text{afm}}$	Eq. (6.32)
5	$T = \frac{p}{2} I_f L_{\text{afm}} I_a$	Eq. (6.34)
6	Effective field	
	$I_{ef} = I_{f_a} \pm \frac{N_s}{N_f} I_s - \text{A.R.}$	Eq. (6.78)
	Two-pulse rectifier	
7	$V_{\text{av}_o} = 2\frac{V_m}{\pi}\cos\alpha$	Eq. (6.83)
	Six-pulse rectifier	
8	$V_{\text{av}} = 3\frac{V_m}{\pi}\cos\alpha$	Eq. (6.86)
9	$I_{\text{rms}} = \sqrt{\frac{2}{3}}\, I_{\text{av}_o}$	Eq. (6.91)
10	$\lvert S \rvert = \sqrt{2} V_{L\text{-}L} I_{\text{av}_o}$	Eq. (6.92)
11	$\cos\theta_T = \frac{3}{\pi}\cos\alpha$	Eq. (6.96)
12	$\cos\theta_T = g\cos\theta$	Eq. (6.98)

TABLE 6-5 Data for a 300 kW, 1150 r/min, 0.95 efficient, 1052 A, 300 V, four-pole, separately excited dc motor.

a. Resistances

Winding(s)	Resistance in ohms at 25°C
Armature	0.005
Commutating and Compensating	0.0045
Separate Field	2.07

(Continued)

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b. Load characteristic

Load in Percent	Armature Voltage (V)	Armature Current (A)	Field Voltage (V)	Field Current (A)	Speed (r/min)	Commutation
0	300	0	55	22.9	1175	Sparkless
25	300	270	55	22.9	1165	Sparkless
50	300	540	55	22.9	1160	Sparkless
75	300	800	55	22.9	1150	Sparkless
100	300	1052	55	22.9	1150	Sparkless
125	300	1350	55	22.9	1150	Sparkless
150	300	1600	55	22.9	1150	Sparkless

c. Temperature rise test

Air supply: 60 m^3/min

Time in Hours	Load	Armature Voltage	Armature Current	Field Voltage	Field Current	Speed (r/min)
2	100%	300 V	1075 A	55 V	22.9 A	1150

	Armature Coil	Armature Core	Armature Commut.	Frame	Field Windings Main	Field Windings Commut.	Field Windings Comp.	Bearings Commutator Side	Bearings Opposite Side
Temp., °C	48	45	55	18	35	33	32	20	15

d. Measurement of air gap

Main pole: 3.5 mm

Interpole: 7.5 mm

Based on data from Siemens Electric Limited

TABLE 6-6 Typical efficiency of dc drives at rated speed

Rating kW	Efficiency Transformer	Efficiency Control Circuit	Efficiency Motor	Efficiency Total
15		0.98	0.92	0.90
30		0.99	0.88	0.87
150	0.97	0.99	0.89	0.85
225	0.98	0.99	0.90	0.87
750	0.99	0.99	0.94	0.92
1500	0.99	0.99	0.94	0.92

Based on data from Siemens Electric Limited

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6.4 Review Questions

1. Describe armature voltage (V_a). Is this voltage ac or dc in waveform? Why is armature voltage also called voltage generated (V_g), back emf (V_b), countervoltage (V_c), or speed voltage?
2. Differentiate between speed voltage and transformer voltage.
3. What is armature reaction, and what are its *two adverse effects* on the operation of dc machines?
4. Under what condition is the armature reaction proportional to the armature current squared?
5. What controls the shape of the magnetization characteristic?
6. What limits the operation of dc machines?
7. What are the *three adverse effects* of high starting current on dc machines?
8. Are the rotating parts of a dc machine the brushes or the commutator segments?
9. At what part of the flux-density distribution is the armature conductor located at the instant when commutation takes place? What is the reason for this?
10. How would you measure the mutual inductance between the armature and field windings of a separately excited motor?
11. Where are the series, the commutating, and the compensating windings located physically? What are their functions?
12. Draw the torque-versus-speed characteristics for series, shunt, and compound motors.
13. Draw the equivalent circuits of long-shunt and short-shunt dc compound machines. Consider differential and cumulative-type connections.
14. Why are dc shunt motors normally equipped with a variable external resistor that is connected in series with the armature?
15. What is the power factor of a six-pulse thyristorized rectifier, as seen from its ac input terminals?
16. What is the average value of the dc voltage for a six-pulse thyristorized rectifier?

6.5 Problems

6-1 At 1800 r/min, a separately excited dc generator with constant excitation develops an open-circuit terminal voltage of 400 V. Determine the electromagnetic torque when the machine operates as a motor and draws an armature current of 60 A. Assume an armature reaction of 5%.

6-2 A 10 kW, 230 V, shunt dc motor has an efficiency of 88% at full-load. The resistances of the armature and shunt field are 0.12 and 230 ohms, respectively. At no-load, the motor rotates at 800 r/min and draws 4 A from the 230 V source.

a. At full-load conditions, determine the armature current, the electromagnetic torque, and the speed of the motor (neglect armature reaction).

b. Repeat (a), assuming an armature reaction of 5%.

6-3 A 200 V, 1200 r/min, 30 A, dc shunt motor drives a ventilating fan whose torque requirement is proportional to the speed squared. The resistances of shunt-field and

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armature windings are 100 and 0.15 ohms, respectively. Neglecting rotational losses and armature reaction, determine:

a. The starting current of the motor.

b. The resistance that must be inserted in series with the armature to produce a motor speed of 1000 r/min.

c. The resistance of the field rheostat that must be connected in series with the field winding to yield a speed of 2000 r/min.

6-4 The generator-motor system shown in Fig. P6-4 is used as a variable-speed drive. In actual setups, the speed of the motor is controlled by varying the field of the generator. For this problem, however, assume that the fields of the motor and generator remain constant. The system's nameplate data and armature reaction are as shown on the diagram. The generator's prime mover is a 3-ϕ induction motor whose slip at no-load is negligible and at full-load is 4%. Estimate:

a. The voltage regulation of the generator.

b. The speed regulation of the motor.

6-5 The open-circuit characteristic of a 220 V dc series motor at 1800 r/min is approximated by

$$V_g = \frac{260\, I_a}{25 + I_a}$$

The motor draws 90 A when it runs loaded at 1800 r/min. If the load conditions change and the motor draws 70 A, determine:

a. The speed.

b. The electromagnetic torque developed.

c. The useful output power if the mechanical losses within the motor consume a torque of 2 N · m.

d. The mathematical relationship that describes the open-circuit characteristic for the condition in (a).

6-6 A short-shunt compound generator has its auxiliary field winding connected in such a way that its magnetic field opposes that of the main field winding. The ratio of the effective number of turns per pole of the main field to auxiliary field winding is 175:1. The armature resistance is 0.10 ohm, the series-field resistance is 0.05 ohm, and the shunt-field resistance is 125 ohms. Data for the open-circuit characteristic at 1800 r/min are as follows:

Armature voltage (V):	50	100	200	260	300	318
Field current (A):	0.21	0.41	1.0	1.51	2.0	2.7

When driven at 1800 r/min, the generator delivers 60 A to a load at 250 V. Determine the speed at which the generator must be driven in order to deliver 75 A at

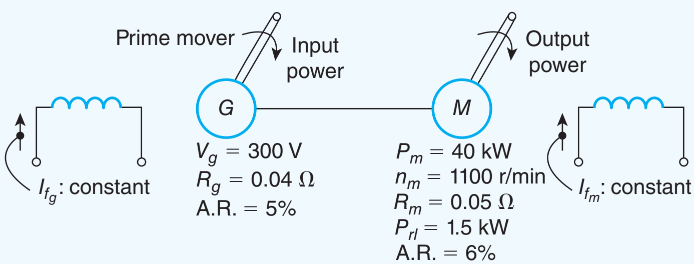

FIG. P6-4

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300 V. Assume that armature reaction is proportional to the armature current.

6-7 A 600 V, 7.46 kW, 1200 r/min series dc motor drives a constant torque load of 50 N · m. The series and armature resistances are 0.10 and 0.20 ohm, respectively. In order to obtain variable speed, a resistance R_x is connected in parallel with the field winding. Neglecting armature reaction and rotational losses,

a. Show that the armature current as a function of R_x is given by

$$I_a = 11.48\sqrt{\frac{R_x + 0.1}{R_x}}$$

b. Show that the speed as a function of R_x is given by

$$I_a = 138.47\sqrt{\frac{R_x + 0.1}{R_x}} - \frac{0.53}{R_x}(1.5R_x + 0.1)$$

c. Sketch the expressions found in (a) and (b).

6-8 In the Ward-Leonard speed control system shown in Fig. P6-8, the armature of the separately excited dc generator is connected directly to that of a separately excited dc motor. The generator is driven at constant speed, and its open-circuit characteristic at the operating region is described by

$$V_g = 5 + 10I_{f_g}$$

The motor has an efficiency of 95%. When the voltage at the terminals of the generator is 320 V, the motor runs at 1200 r/min and delivers 50 kW to the load.

a. For a constant torque-load requirement of 200 N · m, show that the field current of the generator is related to the speed of the motor by

$$I_{f_g} = 0.234\omega + 0.78$$

b. For a constant load-power (50 kW) requirement, show that the field current of the generator is related to the speed of the motor by

$$I_{f_g} = 0.234\omega + 1.97$$

c. Sketch the expressions derived in (a) and (b).

6-9 A single-phase, full-wave rectifier operating at a firing angle of α degrees supplies power to a resistive load. Show that the apparent power factor of the load, as seen from the transformer's secondary terminals, is given by

$$\cos\theta_T = \sqrt{\frac{\pi - \alpha + \dfrac{\sin 2\alpha}{2}}{\pi}}$$

6-10 A separately excited dc motor is supplied from a 208 V source through a thyristorized, single-phase, full-wave

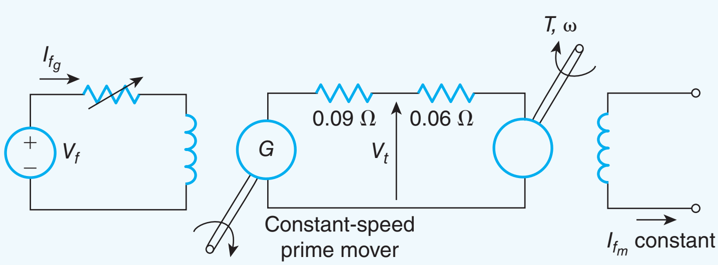

FIG. P6-8

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rectifier. The armature resistance is 0.10 ohm and at nominal operating conditions,

$$n = 800 \text{ rpm}, \qquad I_a = 80 \text{ A}$$

and the thyristor's firing angle is 40 degrees.

By assuming that the field current is constant, sketch the motor's speed as a function of the firing angle when:

a. Constant torque is developed.

b. Constant power is developed.

6-11 A 208 V, separately excited dc motor is controlled through a single-phase, full-wave thyristorized bridge. The armature resistance is 0.1 ohm, and when the speed is 950 rpm, the thyristor's firing angle is 40 degrees and the motor draws 100 A. Sketch the field current as a function of the firing angle while the motor's power is constant.

6-12 As shown in Fig. P6-12, a six-pulse rectifier supplies a 200 kW, 600 V, 0.93 efficient dc motor. The transformer is fed from a 4160 V substation. At nominal operating conditions, the thyristor's firing angle is 36 degrees. Determine:

a. The transformer's turns ratio.

b. The kVA rating of the transformer.

c. The apparent power factor, as seen from the terminals of the substation.

Assume that the transformer is 96% efficient.

6-13 Two dc drives, each 1000 kW, 800 V, and 0.95 efficient, are connected, as shown in Fig. P6-13, to a 4.16 kV substation. Assuming that the motors draw rated current, determine:

a. The thyristor's firing angle.

b. The magnitude of the total current through the 4.16 kV feeder.

c. The apparent power factor, as seen from the secondary lines of one of the transformers.

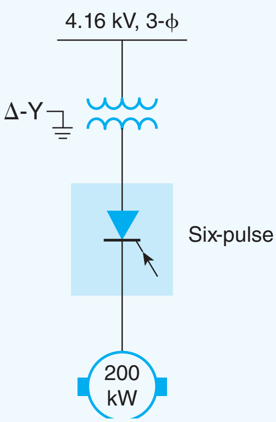

FIG. P6-12

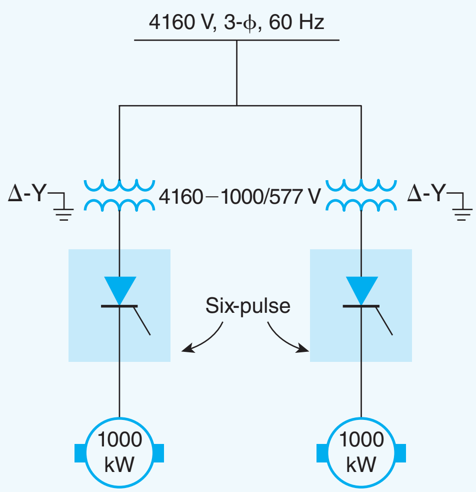

FIG P6-13

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